Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 25, 2002*
Accepted: *October 26, 2002*

(Non-)abelian gauged supergravities in nine dimensions

Eric Bergshoeff, Tim de Wit, Ulf Gran, Román Linares and Diederik Roest

Centre for Theoretical Physics, University of Groningen
Nijenborgh 4, 9747 AG Groningen, The Netherlands
E-mail: e.a.bergshoeff@phys.rug.nl, t.c.de.wit@phys.rug.nl, u.gran@phys.rug.nl, r.linares@phys.rug.nl, d.roest@phys.rug.nl

Abstract: We construct five different two-parameter massive deformations of the unique nine-dimensional $N = 2$ supergravity. All of these deformations have a higher-dimensional origin via Scherk-Schwarz reduction and correspond to gauged supergravities. The gauge groups we encounter are SO(2), $\mathrm{SO}(1,1)^+$, $\mathbb{R}$, $\mathbb{R}^+$ and the two-dimensional non-abelian Lie group $A(1)$, which consists of scalings and translations in one dimension. We make a systematic search for half-supersymmetric domain walls and non-supersymmetric de Sitter space solutions. Furthermore, we discuss which of the supergravities can be considered as candidate low-energy limits of compactified superstring theory.

Keywords: String Duality, Superstring Vacua, Extended Supersymmetry, Supergravity Models.

Contents

1. Introduction

It is well-known that the low-energy limit of superstring theory and/or M-theory is described by a supergravity theory in the same spacetime dimension and with the same number of supersymmetries. Thus, M-theory leads to $D = 11$ supergravity and type-IIA/IIB superstring theory leads to $D = 10$ IIA/IIB supergravity. The same applies to the compactifications of these theories to lower dimensions.

The other way round is less clear: not every supergravity theory has necessarily a string or M-theory origin. A well-known example of a supergravity theory whose role in string theory was unclear until a few years ago is the $D = 10$ massive supergravity theory of Romans [1]. It was pointed out by Romans that the $D = 10$ (massless) IIA supergravity theory [2, 3] can be deformed into a massive supergravity with mass parameter m_{R}. The role of this massive supergravity within string theory has become clear only after the introduction of the D-branes, in particular the D8-brane [4]. An interesting feature of the massive supergravity of Romans is that the lagrangian possesses a dilaton potential proportional to m_{R}^2 which acts as an effective cosmological constant. Due to this scalar potential the massive supergravity, unlike the massless case, does not allow a maximally supersymmetric Minkowski spacetime as a vacuum solution. Instead, the scalar potential leads to the possibility of a half-supersymmetric domain wall solution interpolating between different values of the cosmological constant. Such a solution indeed exists [5, 6] and is identified as the D8-brane of [4].

The massive supergravity of [1] is *not* a gauged supergravity and, at the field theory level, has *no* $D = 11$ origin.[1] The only candidate symmetry of the lagrangian to be gauged is a rigid $\mathbb{R}^+$ symmetry (see table 2). However, the Ramond-Ramond gauge vector has a nontrivial weight under this $\mathbb{R}^+$ symmetry and this leads to inconsistencies with the supersymmetry algebra. There does exist another massive deformation of $D = 10$ IIA supergravity, with mass parameter m_{11}, which *is* a gauged supergravity and *does* have a $D = 11$ origin [8, 9]. However, it can only be defined at the level of the equations of motion. The Ramond-Ramond gauge vector has weight zero with respect to the $\mathbb{R}^+$ group that is gauged (see table 2) and in this case there are no inconsistencies with the supersymmetry algebra. The role of this second massive deformation within string theory is not (yet) clear. An interesting feature of the theory is that it allows for a (non-supersymmetric) de Sitter space solution [9]. The possible physical significance of this de Sitter space solution has been discussed in [10, 11].

A common feature of the $D = 10$ massive supergravity of [1] and the $D = 10$ gauged supergravity of [8, 9] is that there is a dilaton potential which is proportional to the square of the mass parameter, m_{R}^2 and m_{11}^2, respectively. Due to this scalar potential the $D = 10$ Minkowski spacetime is no longer a maximally supersymmetric vacuum solution of the theory. Instead one can look for half-supersymmetric vacuum solutions. A natural class of half-supersymmetric solutions that makes use of the scalar potential is the set of domain-wall solutions, like the D8-brane mentioned above. Recently, domain wall solutions of lower-dimensional supergravities have attracted attention in view of their

[1] We assume that we are not using the existence of extra Killing vectors, like in [7].

relevance for a supersymmetric Randall-Sundrum scenario [12, 13], the domain-wall/QFT correspondence [14, 15] and applications to cosmology [16, 17]. In all these applications the properties of the domain walls play a crucial role and these properties are determined by the details of the scalar potential.

Motivated by this we studied in a previous paper general domain wall solutions in $D = 9$ dimensions [18].[2] We took $D = 9$ because on the one hand this case shares some of the complexities of the lower-dimensional cases, on the other hand the scalar potential for this case is simple enough to study the corresponding domain-wall solutions in full detail. The supergravity theory we considered in [18] was obtained by a generalized Scherk-Schwarz reduction of $D = 10$ IIB supergravity. This is not the most general possibility in $D = 9$. The aim of this paper is to make a systematic search for massive deformations of the unique $D = 9$, N=2 supergravity theory. All deformations we find correspond to *gauged* supergravities. Such supergravities have a gauge symmetry which reduces, for constant values of the gauge parameter, to a nontrivial rigid symmetry. The hope is that the $D = 9$ case will teach us something about the more complicated situation in $D < 9$ dimensions.

In the first part of this paper we will present in two steps the $D = 9$ gauged supergravities we have found. In a first step we will present seven massive deformations with a *single* mass parameter m, all giving rise to gauged supergravities. All of them are obtained by generalized dimensional reduction [22] from a higher-dimensional theory (11D, IIA or IIB supergravity). The consistency of these 9D gauged supergravities is guaranteed by their higher-dimensional origin. The gauge groups we encounter are either[3] $\mathrm{SO}(2)$, $\mathrm{SO}(1,1)^+$, $\mathbb{R}$ (all of which are subgroups of $\mathrm{SL}(2,\mathbb{R})$ with invariant metrics $\mathrm{diag}(1,1)$, $\mathrm{diag}(1,-1)$ and $\mathrm{diag}(1,0)$, respectively [23, 24]), $\mathbb{R}^+$ or the two-dimensional non-abelian Lie group $\mathrm{A}(1)$.[4] The latter is the affine group of the line and consists of so-called collinear transformations (scalings and translations) in one dimension and forms a non-semi-simple Lie group [25, 26].

In a second step we will consider combinations of these seven massive deformations. The closure of the supersymmetry algebra will be guaranteed based on a linearity argument but it turns out that non-linear restrictions enter via the back door. Satisfying these restrictions leaves us with five different two-parameter deformations (rather than the seven-parameter deformation that one could have if there were no non-linear restrictions). These are the most general gauged supergravities we construct in this paper.

In the second part of this paper we make a systematic search for vacuum solutions of the supergravities we have obtained. The existence of such vacuum solutions is needed in order to define the spectrum of the theory as fluctuations around this vacuum. Our search includes half-supersymmetric domain wall solutions and non-supersymmetric de Sitter spaces.

Throughout the paper we will reduce field equations rather than lagrangians. The

[2]For earlier discussions of domain wall solutions in $D = 9$ dimensions, see [19, 20]. For a more recent discussion, see [21].

[3]Throughout the paper we will use the notation $\mathrm{SO}(1,1)^+$ rather than the (isomorphic) $\mathbb{R}^+$ for the scaling symmetry that is a subgroup of $\mathrm{SL}(2,\mathbb{R})$. The different notation is used to denote the different origin.

[4]This is unrelated to the A-D-E classification of simple Lie groups.

JHEP10(2002)061

reason for this is the fact that (some of) the rigid symmetries we employ for Scherk-Schwarz reduction scale the lagrangian. As was noted by [9] and is illustrated by the SS reduction of a simple toy model in appendix B, Scherk-Schwarz reduction with a symmetry that scales the lagrangian can only be performed at the level of the field equations. Reduction of the lagrangian itself gives rise to the wrong equations. In fact, the reduced field equations can not be obtained as Euler-Lagrange equations of *any* lagrangian.

This paper is organized as follows. In section 2 we briefly review the situation in $D = 11$ dimensions where no massive deformation has been constructed so far. This case is needed for the discussion of the $D = 10$ and $D = 9$ cases. In section 3 we discuss the two different massive deformations of $D = 10$ IIA supergravity. In section 4 we review the case of $D = 10$ IIB supergravity. This case does not allow massive deformations but will be needed for the discussion of the $D = 9$ case. In section 5 we present 7 massive deformations of the maximally supersymmetric $D = 9$ supergravity theory. They all are gauged supergravities with gauge group $\mathrm{SO}(2)$, $\mathrm{SO}(1,1)^+$, $\mathbb{R}$, $\mathbb{R}^+$ or $\mathrm{A}(1)$. In section 6 we show, by combining the different gauged supergravities, that there exist five different two-parameter massive supergravity theories. In section 7 we make a systematic search for vacuum solutions of the supergravities we have obtained. Finally, in section 8 we give our conclusions. In particular, we discuss which of the $D = 9$ supergravities can be considered as candidate low-energy limits of (compactified) superstring theory. We give four appendices. Appendix A contains our conventions. Appendix B discusses the Scherk-Schwarz reduction of a dilaton-gravity toy model. Appendix C contains the supersymmetry transformations of *massless* $D = 11$, 10 and 9 supergravity plus the reduction Ansätze to go from $D = 11$ to $D = 10$ to $D = 9$. Finally, in appendix D we discuss some manipulations with spinors and gamma-matrices in ten and nine dimensions.

2. $D = 11$ supergravity

We first consider eleven-dimensional supergravity. Its field content is given by[5]

$$D = 11: \qquad \left\{ \hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}}, \hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}}, \hat{\hat{\psi}}_{\hat{\hat{\mu}}} \right\}. \tag{2.1}$$

The $D = 11$ supersymmetry transformations are given in appendix C, see eq. (C.1). These supersymmetry rules are covariant under an $\mathbb{R}^+$ symmetry with parameter $\hat{\hat{\alpha}}$ [27]. The weights of the $D = 11$ fields under this $\mathbb{R}^+$ are given in table 1. Note that the lagrangian is not invariant but scales with weight $w = 9$. Therefore this $\mathbb{R}^+$ is a symmetry of the equations of motion only.

$\mathbb{R}^+$	$\hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}}$	$\hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}}$	$\hat{\hat{\psi}}_{\hat{\hat{\mu}}}$	$\hat{\hat{\epsilon}}$	$\hat{\hat{\mathcal{L}}}$
$\hat{\hat{\alpha}}$	1	3	$\frac{1}{2}$	$\frac{1}{2}$	9

Table 1: The $\mathbb{R}^+$-weights of the $D = 11$ supergravity fields, the supersymmetry parameters $\hat{\hat{\epsilon}}$ and the lagrangian $\hat{\hat{\mathcal{L}}}$.

No massive deformation of the eleven-dimensional supergravity theory is known. In particular, no cosmological constant can be added [28]. One problem with a $D = 11$ supersymmetric cosmological constant is that its reduction gives rise to a $D = 10$ cosmological

[5]In order to distinguish between $D = 11$, $D = 10$ and $D = 9$ we indicate $D = 11$ fields and indices with a double hat, $D = 10$ fields and indices with a single hat and $D = 9$ fields and indices with no hat.

$\mathbb{R}^+$	$\hat{e}_{\hat{\mu}}{}^{\hat{a}}$	$\hat{B}_{\hat{\mu}\hat{\nu}}$	$e^{\hat{\phi}}$	$\hat{A}_{\hat{\mu}}$	$\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$	$\hat{\psi}_{\hat{\mu}}$	$\hat{\lambda}$	$\hat{\epsilon}$	$\hat{\mathcal{L}}$	Origin
$\hat{\alpha}$	$\frac{9}{8}$	3	$\frac{3}{2}$	0	3	$\frac{9}{16}$	$-\frac{9}{16}$	$\frac{9}{16}$	9	$\hat{\hat{\alpha}}$
$\hat{\beta}$	0	$\frac{1}{2}$	1	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0	0	0	

Table 2: The $\mathbb{R}^+$-weights of the $D = 10$ IIA supergravity fields, the supersymmetry parameter $\hat{\epsilon}$ and the lagrangian $\hat{\mathcal{L}}$.

constant with a dilaton coupling that differs from Romans' massive deformation. A general deformation of $D = 11$ supergravity involving the use of extra Killing vectors has been considered in [29]. We will not consider this possibility in this paper.

3. Massive deformations of $D = 10$ IIA supergravity

A Kaluza-Klein reduction of the eleven-dimensional theory yields the IIA theory in ten dimensions.[6] The field content of the $D = 10$ IIA supergravity theory is given by

$$D = 10 \text{ IIA}: \qquad \left\{\hat{e}_{\hat{\mu}}{}^{\hat{a}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{\phi}, \hat{A}_{\hat{\mu}}, \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{\psi}_{\hat{\mu}}, \hat{\lambda}\right\}. \tag{3.1}$$

The supersymmetry transformations rules are given in eq. (C.5). For later purposes we indicate these (undeformed) supersymmetry transformations by δ_0. The transformation rules have two $\mathbb{R}^+$-symmetries, one with parameter $\hat{\alpha}$ that scales the lagrangian and one with parameter $\hat{\beta}$ that leaves the lagrangian invariant. The first symmetry follows via dimensional reduction from the $D = 11$ $\mathbb{R}^+$-symmetry with parameter $\hat{\hat{\alpha}}$. The weights of these two $\mathbb{R}^+$-symmetries are given in table 2. The gauge symmetry associated to the Ramond-Ramond vector, with parameter $\hat{\lambda}$, reads

$$\hat{A} \to \hat{A} - \mathrm{d}\hat{\lambda}\,, \qquad \hat{C} \to \hat{C} - \mathrm{d}\hat{\lambda}\,\hat{B}\,. \tag{3.2}$$

The $D = 10$ IIA supergravity theory allows two massive deformations which we discuss one by one below.

3.1 Deformation m_{R}: $D = 10$ massive supergravity

The first massive deformation, with mass parameter m_{R}, is due to Romans [1]. In this case (the same is true for all other cases) the supersymmetry transformations receive two types of massive deformations: explicit and implicit ones. The explicit deformations are terms, at most linear in m_{R}, that are added to the original supersymmetry rules. These explicit deformations are denoted by $\delta_{m_{\mathrm{R}}}$ and are given, in terms of a superpotential $W(\hat{\phi})$ and derivatives thereof, by

$$m_{\mathrm{R}}: \begin{cases} \delta_{m_{\mathrm{R}}}\hat{\psi}_{\hat{\mu}} = -\frac{1}{8}W\hat{\Gamma}_{\hat{\mu}}\hat{\epsilon}\,, \qquad \text{with } W = \frac{1}{4}e^{5\hat{\phi}/4}m_{\mathrm{R}}\,, \\ \delta_{m_{\mathrm{R}}}\hat{\lambda} = 4\frac{\delta W}{\delta\hat{\phi}}\hat{\epsilon}\,. \end{cases} \tag{3.3}$$

There are further implicit massive deformations to the original supersymmetry rules δ_0,

[6] We have used the reduction Ansätze (C.4) with $m_{11} = 0$.

which are given in eq. (C.5), due to the fact that in these rules one must replace all field strengths by corresponding *massive* field strengths which are given by

$$\hat{F} = \mathrm{d}\hat{A} + m_{\mathrm{R}}\hat{B}\,, \qquad \hat{H} = \mathrm{d}\hat{B}\,, \qquad \hat{G} = \mathrm{d}\hat{C} + \hat{A}\hat{H} + \frac{1}{2}m_{\mathrm{R}}\hat{B}\hat{B}\,. \tag{3.4}$$

The lagrangian contains terms linear and quadratic in m_{R}. Again there are implicit deformations, via the massive field strengths, and explicit deformations. The explicit deformations quadratic in the mass parameter define the scalar potential which can be written in terms of the superpotential $W(\hat{\phi})$ and derivatives thereof.

Requiring closure of the supersymmetry algebra one finds the linear deformations of the fermionic (gravitino and dilatino) field equations in Roman's theory:

$$m_{\mathrm{R}}: \begin{cases} X_{m_{\mathrm{R}}}(\hat{\psi}^{\hat{\mu}}) \equiv m_{\mathrm{R}} e^{5\hat{\phi}/4}\hat{\Gamma}^{\hat{\mu}\hat{\nu}}(\frac{1}{4}\hat{\psi}_{\hat{\nu}} + \frac{5}{288}\hat{\Gamma}_{\hat{\nu}}\hat{\lambda})\,, \\ X_{m_{\mathrm{R}}}(\hat{\lambda}) \equiv m_{\mathrm{R}} e^{5\hat{\phi}/4}\hat{\Gamma}^{\hat{\nu}}(-\frac{5}{4}\hat{\psi}_{\hat{\nu}} - \frac{21}{160}\hat{\Gamma}_{\hat{\nu}}\hat{\lambda})\,. \end{cases} \tag{3.5}$$

The undeformed equations, $X_0(\hat{\psi}^{\hat{\mu}})$ and $X_0(\hat{\lambda})$, are given in eqs. (C.7).

Under supersymmetry the fermionic field equations, $X_0 + X_{m_{\mathrm{R}}}$, transform into the deformed bosonic equations of motion. Since we will only be interested in finding solutions that are carried by the metric and the scalars it is convenient to truncate away all bosonic fields except the metric and the dilaton.[7] After this truncation we find that under supersymmetry the fermionic field equations transform into

$$\begin{aligned} (\delta_0 + \delta_{m_{\mathrm{R}}})(X_0 + X_{m_{\mathrm{R}}})(\hat{\psi}^{\hat{\mu}}) =& \frac{1}{2}\hat{\Gamma}^{\hat{\nu}}\hat{\epsilon}\Big[\hat{R}^{\hat{\mu}}{}_{\hat{\nu}} - \frac{1}{2}\hat{R}\hat{g}^{\hat{\mu}}{}_{\hat{\nu}} - \frac{1}{2}(\partial^{\hat{\mu}}\hat{\phi})(\partial_{\hat{\nu}}\hat{\phi}) + \\ & + \frac{1}{4}(\partial\hat{\phi})^2\hat{g}^{\hat{\mu}}{}_{\hat{\nu}} + \frac{1}{4}m_{\mathrm{R}}^2 e^{5\hat{\phi}/2}\hat{g}^{\hat{\mu}}{}_{\hat{\nu}}\Big]\,, \\ (\delta_0 + \delta_{m_{\mathrm{R}}})(X_0 + X_{m_{\mathrm{R}}})(\hat{\lambda}) =& \hat{\epsilon}\Big[\Box\hat{\phi} - \frac{5}{4}m_{\mathrm{R}}^2 e^{5\hat{\phi}/2}\Big]\,. \end{aligned} \tag{3.6}$$

At the right-hand side we thus find the Romans' bosonic field equations for the metric and the dilaton, one solution of which is the D8-brane. Note that the bosonic field equations contain terms quadratic in the mass parameter.

Romans' theory is not known to have a higher-dimensional supergravity origin. Neither is it a gauged supergravity. A candidate symmetry of the lagrangian to be gauged is the $\hat{\beta}$ symmetry of table 2. However, the candidate gauge field $\hat{A}_{\hat{\mu}}$ has a nontrivial weight under $\hat{\beta}$. This means that the curl $d\hat{A}$ transforms with a non-covariant term proportional to $d\hat{\lambda}\hat{A}$. Such a term cannot be cancelled by adding an extra term, such as $\hat{B}$, to the definition of the $\hat{A}$ curvature. In short, the $\hat{\beta}$ symmetry cannot be gauged [3]. The same table shows that on the other hand $\hat{A}_{\hat{\mu}}$ has weight zero under the $\hat{\alpha}$-symmetry which is a symmetry of the equations of motion only. This $\hat{\alpha}$-symmetry can indeed be gauged at the level of the equations of motion. This gauging leads to the $D = 10$ gauged supergravity discussed below.

3.2 Deformation m_{11}: $D = 10$ gauged supergravity

The second massive deformation, with mass parameter m_{11}, has been considered in [8, 9] and is a gauged supergravity. It can be obtained by generalized Scherk-Schwarz reduction of

[7] Note that a further truncation to $\phi = c$ is inconsistent.

JHEP10(2002)061

$D=11$ supergravity using the $\mathbb{R}^+$ symmetry $\hat{\hat{\alpha}}$ of table 1 [9]. The corresponding reduction Ansätze, with $m_{11}\neq 0$, are given in eq. (C.4). This reduction leads to the following explicit massive deformations of the $D=10$ IIA supersymmetry rules:

$$m_{11}: \begin{cases} \delta_{m_{11}}\hat{\psi}_{\hat{\mu}} = \tfrac{9}{16}m_{11}e^{-3\hat{\phi}/4}\hat{\Gamma}_{\hat{\mu}}\Gamma_{11}\hat{\epsilon}\,, \\ \delta_{m_{11}}\hat{\lambda} = \tfrac{3}{2}m_{11}e^{-3\hat{\phi}/4}\Gamma_{11}\hat{\epsilon}\,. \end{cases} \tag{3.7}$$

The implicit massive deformations of the original supersymmetry rules δ_0 are given by the massive bosonic field strengths

$$\mathrm{D}\hat{\phi} = \mathrm{d}\hat{\phi} + \frac{3}{2}m_{11}\hat{A}\,, \qquad \hat{F} = \mathrm{d}\hat{A}\,, \qquad \hat{H} = \mathrm{d}\hat{B} + 3m_{11}\hat{C}\,, \qquad \hat{G} = \mathrm{d}\hat{C} + \hat{A}\hat{H}\,, \tag{3.8}$$

while the covariant derivative of the supersymmetry parameter is given by

$$D_{\hat{\mu}}\hat{\epsilon} = \left(\partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}} + \frac{9}{16}m_{11}\hat{\Gamma}_{\hat{\mu}}\hat{A}\right)\hat{\epsilon}\,. \tag{3.9}$$

The gauge vector in the definition of the covariant derivative is required to make the derivative of the supersymmetry parameter *and* the spin connection $\mathbb{R}^+$-covariant.

The linear deformations of the fermionic field equations read in this case

$$m_{11}: \begin{cases} X_{m_{11}}(\hat{\psi}^{\hat{\mu}}) \equiv m_{11}e^{-3\hat{\phi}/4}\Gamma_{11}\hat{\Gamma}^{\hat{\mu}\hat{\nu}}(-\tfrac{9}{2}\hat{\psi}_{\hat{\nu}} + \tfrac{17}{48}\hat{\Gamma}_{\hat{\nu}}\hat{\lambda})\,, \\ X_{m_{11}}(\hat{\lambda}) \equiv m_{11}e^{-3\hat{\phi}/4}\Gamma_{11}\hat{\Gamma}^{\hat{\nu}}(\tfrac{3}{2}\hat{\psi}_{\hat{\nu}} - \tfrac{9}{16}\hat{\Gamma}_{\hat{\nu}}\hat{\lambda})\,. \end{cases} \tag{3.10}$$

We first consider the truncation that all bosonic fields except the metric and the dilaton are set equal to zero. Under supersymmetry the fermionic field equations transform into

$$\begin{aligned} (\delta_0+\delta_{m_{11}})(X_0+X_{m_{11}})(\hat{\psi}^{\hat{\mu}}) =& \frac{1}{2}\hat{\Gamma}^{\hat{\nu}}\hat{\epsilon}\Big[\hat{R}^{\hat{\mu}}{}_{\hat{\nu}} - \frac{1}{2}\hat{R}\hat{g}^{\hat{\mu}}{}_{\hat{\nu}} - \frac{1}{2}(\partial^{\hat{\mu}}\hat{\phi})(\partial_{\hat{\nu}}\hat{\phi}) + \frac{1}{4}(\partial\hat{\phi})^2\hat{g}^{\hat{\mu}}{}_{\hat{\nu}} + \\ & + 36m_{11}^2e^{-3\hat{\phi}/2}\hat{g}^{\hat{\mu}}{}_{\hat{\nu}}\Big] + \Gamma_{11}\hat{\epsilon}\Big[3m_{11}e^{-3\hat{\phi}/4}\partial^{\hat{\mu}}\hat{\phi}\Big]\,, \\ (\delta_0+\delta_{m_{11}})(X_0+X_{m_{11}})(\hat{\lambda}) =& \hat{\epsilon}\,[\Box\hat{\phi}] + \hat{\Gamma}^{\hat{\nu}}\Gamma_{11}\hat{\epsilon}\Big[9m_{11}e^{-3\hat{\phi}/4}\partial_{\hat{\nu}}\hat{\phi}\Big]\,. \end{aligned} \tag{3.11}$$

The terms involving Γ_{11} are part of the vector field equation. Therefore, to obtain a consistent truncation, we must further truncate the dilaton to zero. One is then left with only the metric satisfying the Einstein equation with a positive cosmological constant, a solution of which is 10D de Sitter space [9].

The reduced theory is a gauged supergravity where the $\mathbb{R}^+$ symmetry $\hat{\alpha}$ of table 2 has been gauged. In particular, the gauge parameter and transformation of the Ramond-Ramond potentials read as follows:[8]

$$\hat{\alpha}: \qquad \Lambda = e^{w_{\hat{\alpha}}m_{11}\hat{\lambda}} \qquad \text{with} \qquad \hat{A}\to\hat{A}-\mathrm{d}\hat{\lambda}\,, \qquad \hat{C}\to e^{3m_{11}\hat{\lambda}}(\hat{C}-\mathrm{d}\hat{\lambda}\,\hat{B})\,, \tag{3.12}$$

where $w_{\hat{\alpha}}$ are the weights under $\hat{\alpha}$. We note that one can take two different limits of the $\hat{\alpha}$ gauge transformations. First, the limit $m_{11}\to 0$ leads to the massless gauge transformations (3.2). Note that $\hat{C}$ transforms trivially under this gauge symmetry in the sense that $\hat{C}$ can be made gauge-invariant after a simple field-redefinition. Secondly, one can take the limit that $\hat{\alpha}$ is constant. This leads to the ungauged $\mathbb{R}^+$ $\hat{\alpha}$-symmetry of table 2.

[8]It is understood that each field with $w_{\hat{\alpha}}\neq 0$ is multiplied by Λ.

A noteworthy feature of the $D = 10$ gauged supergravity is that no lagrangian can be defined for it. In the search for supersymmetric domain wall solutions in $D = 5$ dimensions many other examples of gauged supergravity theories without a lagrangian have been given [30]. Note that one can write down a lagrangian for the ungauged theory. The reason that one cannot write down a lagrangian after gauging is that the symmetry that is gauged is not a symmetry of the lagrangian but only of the equations of motion. It would be instructive to construct the $D = 10$ gauged supergravity from the ungauged theory by gauging the $\hat{\alpha}$-symmetry. Apparently, it shows that one can gauge symmetries that leave a lagrangian invariant up to a scale factor.

4. $D = 10$ IIB supergravity

The other ten-dimensional supergravity theory is chiral IIB. Its field content is

$$D = 10 \text{ IIB:} \qquad \left\{\hat{e}_{\hat{\mu}}{}^{\hat{a}}, \hat{\phi}, \hat{\chi}, \hat{B}^{(1)}_{\hat{\mu}\hat{\nu}}, \hat{B}^{(2)}_{\hat{\mu}\hat{\nu}}, \hat{D}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}, \hat{\psi}_{\hat{\mu}}, \hat{\lambda}\right\}. \tag{4.1}$$

The supersymmetry variations are given in eq. (C.10). The IIB supersymmetry rules transform covariant under the $\mathrm{SL}(2,\mathbb{R})$ transformations (omitting indices):

$$\begin{aligned}
&\hat{\tau} \to \frac{a\hat{\tau} + b}{c\hat{\tau} + d}, \qquad \vec{\hat{B}} \to \Omega\vec{\hat{B}}, \qquad \hat{D} \to \hat{D}, \qquad \text{with } \Omega = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R}), \\
&\hat{\psi} \to \left(\frac{c\,\hat{\tau}^* + d}{c\,\hat{\tau} + d}\right)^{1/4} \hat{\psi}, \qquad \hat{\lambda} \to \left(\frac{c\,\hat{\tau}^* + d}{c\,\hat{\tau} + d}\right)^{3/4} \hat{\lambda}, \qquad \hat{\epsilon} \to \left(\frac{c\,\hat{\tau}^* + d}{c\,\hat{\tau} + d}\right)^{1/4} \hat{\epsilon}.
\end{aligned} \tag{4.2}$$

We have used here the vector notation $\vec{\hat{B}} = \big(\hat{B}^{(1)}, \hat{B}^{(2)}\big)^T$. The group $\mathrm{SL}(2,\mathbb{R})$ contains a set of three one-parameter conjugacy classes defining one compact and two non-compact subgroups. Since they are needed later we will describe them shortly. Each of the subgroups is generated by a $\mathrm{SL}(2,\mathbb{R})$ group element Ω with $\det\Omega = 1$.

1. One non-compact subgroup $\mathbb{R}$ is generated by

$$\Omega_p = e^{\frac{1}{2}\hat{\zeta}\,(\sigma_1 + i\sigma_2)} = \begin{pmatrix} 1 & \hat{\zeta} \\ 0 & 1 \end{pmatrix}. \tag{4.3}$$

   Each element defines a parabolic conjugacy class with $\mathrm{Tr}\,\Omega = 2$. These parabolic transformations leave the combination $(\hat{B}^{(2)})^2$ invariant. Therefore the invariant metric is given by $\mathrm{diag}(0,1)$. The action of the $\mathbb{R}$ $\hat{\zeta}$-symmetry on the fields can not be expressed by assigning weights to the standard basis of fields given in (4.1).

2. An $\mathrm{SO}(1,1)^+$ subgroup which is generated by elements

$$\Omega_h = e^{\hat{\gamma}\,\sigma_3} = \begin{pmatrix} e^{\hat{\gamma}} & 0 \\ 0 & e^{-\hat{\gamma}} \end{pmatrix}. \tag{4.4}$$

   Each element defines a hyperbolic conjugacy class with $\mathrm{Tr}\,\Omega > 2$. These hyperbolic transformations leave the combination $\hat{B}^{(1)}\hat{B}^{(2)}$ invariant. After diagonalization this leads to an invariant metric given by $\mathrm{diag}(1,-1)$. The weights corresponding to the $\mathrm{SO}(1,1)^+$ $\hat{\gamma}$-symmetry are given in table 3.

JHEP10(2002)061

$\mathbb{R}^+$	$\hat{e}_{\hat{\mu}}{}^{\hat{a}}$	$e^{\hat{\phi}}$	$\hat{\chi}$	$\hat{B}^{(1)}_{\hat{\mu}\hat{\nu}}$	$\hat{B}^{(2)}_{\hat{\mu}\hat{\nu}}$	$\hat{D}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$	$\hat{\psi}_{\hat{\mu}}$	$\hat{\lambda}$	$\hat{\epsilon}$	$\hat{\mathcal{L}}$	symmetry
$\hat{\gamma}$	0	-2	2	1	-1	0	0	0	0	0	$\mathrm{SO}(1,1)^+$
$\hat{\delta}$	1	0	0	2	2	4	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	8	$\mathbb{R}^+$

Table 3: The scaling weights of the $D = 10$ IIB supergravity fields, the supersymmetry parameter $\hat{\epsilon}$ and the lagrangian $\hat{\mathcal{L}}$.

3. There is an SO(2) subgroup which is generated by elements Ω of $\mathrm{SL}(2,\mathbb{R})$ with

$$\Omega_e = e^{i\hat{\theta}\,\sigma_2} = \begin{pmatrix} \cos\hat{\theta} & \sin\hat{\theta} \\ -\sin\hat{\theta} & \cos\hat{\theta} \end{pmatrix}. \tag{4.5}$$

Each element defines an elliptic conjugacy class with $\mathrm{Tr}\,\Omega < 2$. The elliptic transformations leave $(\hat{B}^{(1)})^2 + (\hat{B}^{(2)})^2$ invariant. After diagonalization this leads to an invariant metric given by $\mathrm{diag}(1,1)$. The action of the SO(2) $\hat{\theta}$-symmetry on the fields can not be expressed by assigning weights to the standard real basis of fields given in (4.1).

Table 3 contains the weights of the $\hat{\gamma}$-symmetry defined above[9] and of a new $\mathbb{R}^+$ symmetry $\hat{\delta}$ which is *not* a subgroup of $\mathrm{SL}(2,\mathbb{R})$ and that does not leave the lagrangian invariant. One could combine $\mathrm{SL}(2,\mathbb{R})$ with this new $\mathbb{R}^+$ into a $\mathrm{GL}(2,\mathbb{R})$ symmetry that leaves the IIB equations of motion invariant. Its action is the product of the two separate transformations: $\tilde{\Omega} = \Omega\Lambda_{\hat{\delta}}$. This exhausts all the symmetries of $D = 10$ IIB supergravity.

The IIB supergravity theory is not known to have massive deformations. One of the reasons for this is that there is no candidate vector field like in the IIA case.

5. Massive deformations of $D = 9$, $N = 2$ supergravity

The Kaluza-Klein reduction of either (massless) IIA or IIB supergravity gives the unique $D = 9$, $N = 2$ massless supergravity theory. Its field content is given by

$$D = 9: \qquad \left\{e_\mu{}^a, \phi, \varphi, \chi, A_\mu, A^{(1)}_\mu, A^{(2)}_\mu, B^{(1)}_{\mu\nu}, B^{(2)}_{\mu\nu}, C_{\mu\nu\rho}, \psi_\mu, \lambda, \tilde{\lambda}\right\}. \tag{5.1}$$

The supersymmetry rules are given in eq. (C.16). The massless 9-dimensional theory inherits several global symmetries from its parents: two $\mathbb{R}^+$ symmetries α, β from IIA supergravity and one $\mathbb{R}^+$ symmetry δ plus a full $\mathrm{SL}(2,\mathbb{R})$ symmetry from IIB supergravity. The latter leads in particular to an SO(2) symmetry θ, an $\mathrm{SO}(1,1)^+$ symmetry γ and an $\mathbb{R}$-symmetry ζ. The weights of all these symmetries, except for the SO(2) θ-symmetry and $\mathbb{R}$ ζ-symmetry, and their higher-dimensional origin are given in table 4 (see also [27]).

It turns out that only three out of the four scalings given in table 4 are linearly independent. There is a relation

$$\frac{4}{9}\alpha - \frac{8}{3}\beta = \gamma + \frac{1}{2}\delta\,. \tag{5.2}$$

[9] The other two symmetries defined above cannot be defined in terms of weights of real fields only.

$\mathbb{R}^+$	${e_\mu}^a$	e^ϕ	e^φ	χ	A_μ	$A_\mu^{(1)}$	$A_\mu^{(2)}$	$B_{\mu\nu}^{(1)}$	$B_{\mu\nu}^{(2)}$	$C_{\mu\nu\rho}$	ψ_μ	λ	$\tilde{\lambda}$	ϵ	$\mathcal{L}$	Origin
α	$\frac{9}{7}$	0	$\frac{6}{\sqrt{7}}$	0	3	0	0	3	3	3	$\frac{9}{14}$	$-\frac{9}{14}$	$-\frac{9}{14}$	$\frac{9}{14}$	9	11D
β	0	$\frac{3}{4}$	$\frac{\sqrt{7}}{4}$	-$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	0	0	0	0	0	IIA
γ	0	-2	0	2	0	1	-1	1	-1	0	0	0	0	0	0	IIB
δ	$\frac{8}{7}$	0	$-\frac{4}{\sqrt{7}}$	0	0	2	2	2	2	4	$\frac{4}{7}$	$-\frac{4}{7}$	$-\frac{4}{7}$	$\frac{4}{7}$	8	IIB

Table 4: The scaling weights of the 9 dimensional supergravity fields, the supersymmetry parameter ϵ and the lagrangian $\mathcal{L}$.

We observe the following pattern. Using (5.2) to eliminate one of the scaling-symmetries we are left with three independent scaling-symmetries. Each of the three gauge fields $A_\mu, A_\mu^{(1)}, A_\mu^{(2)}$ has weight zero under *two* (linear combinations) of these three symmetries: one is a symmetry of the action, the other is a symmetry of the equations of motion only.

The $D = 9$ SL$(2,\mathbb{R})$ symmetry acts in the following way:

$$\begin{aligned}
&\tau \to \frac{a\tau + b}{c\tau + d}\,, \qquad \vec{A} \to \Omega\vec{A}\,, \qquad \vec{B} \to \Omega\vec{B}\,, \qquad \text{with } \Omega = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R})\,, \\
&\psi_\mu \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{1/4} \psi_\mu\,, \qquad \lambda \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{3/4} \lambda\,, \\
&\tilde{\lambda} \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{-1/4} \tilde{\lambda}\,, \qquad \epsilon \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{1/4} \epsilon\,,
\end{aligned} \tag{5.3}$$

while φ and C are invariant. We have used a vector notation for the two vectors and two antisymmetric tensors, like in $D = 10$. Again one can combine SL$(2,\mathbb{R})$ with an $\mathbb{R}^+$ symmetry to form GL$(2,\mathbb{R})$ with parameter $\tilde{\Omega} = \Omega\Lambda_{\mathbb{R}^+}$.

In addition to the global symmetries there is a number of local symmetries. In particular, the gauge transformations of the vectors read

$$\begin{aligned}
A^{(1)} &\to A^{(1)} - \mathrm{d}\lambda^{(1)}\,, \qquad & A^{(2)} &\to A^{(2)} - \mathrm{d}\lambda^{(2)}\,, \\
A &\to A - \mathrm{d}\lambda\,, & \vec{B} &\to \vec{B} - \vec{A}\,\mathrm{d}\lambda\,.
\end{aligned} \tag{5.4}$$

We now turn to massive deformations of the 9D theory. Applying a SS dimensional reduction of the higher-dimensional supergravities we obtain a number of massive deformations in nine dimensions, as illustrated in figure 1. By employing the different global symmetries of 11D, IIA and IIB supergravity we obtain seven deformations of the unique $D = 9$ supergravity.

Note that the different massive deformations can be related. Symmetries of the massless theory become field redefinitions in the massive theory that only act on the massive deformations. This means that the mass parameters transform under such transformations: they have a scaling weight under the different scaling symmetries and fall in multiplets of SL$(2,\mathbb{R})$. In table 5 the multiplet structure of the massive deformations under SL$(2,\mathbb{R})$ is given. The mass parameter $\tilde{m}_4$ is defined as the S-dual partner of m_4 and can not be obtained by a SS reduction of IIA supergravity.

JHEP10(2002)061

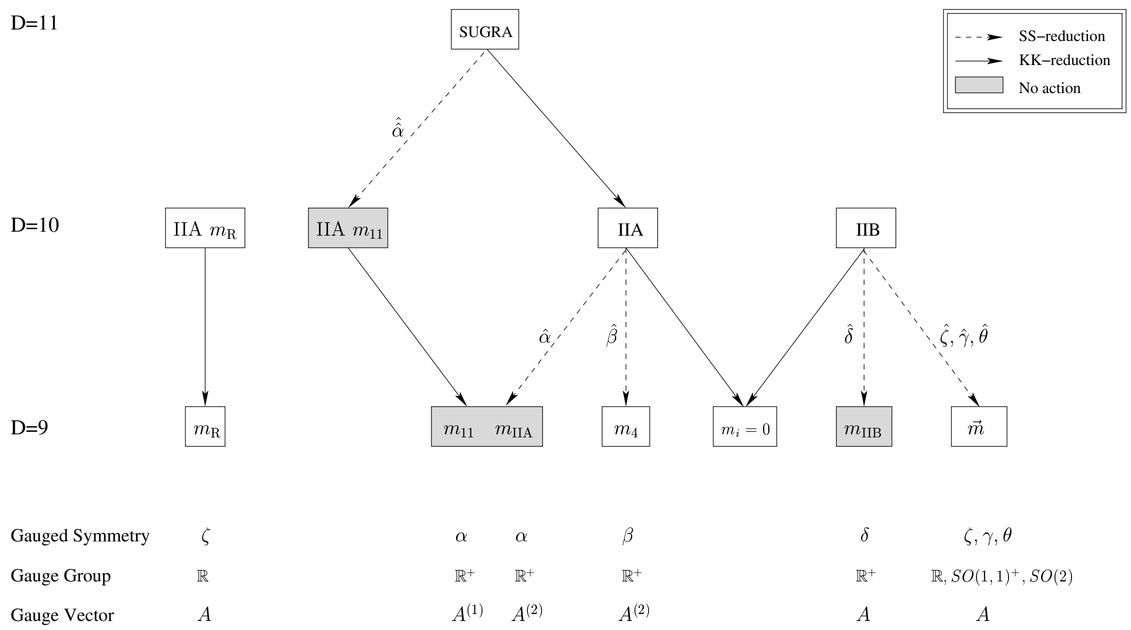

Figure 1: Overview of all reductions performed in this paper. These cases can all be interpreted as gauged supergravities, with gauged symmetry and corresponding gauge field as given in the figure. Mass parameters in the same box, such as $m_{11}, m_{\rm IIA}$ or m_1, m_2, m_3, form a multiplet under $\mathrm{SL}(2,\mathbb{R})$. Further details of these cases will be given below. Note that the two ways of obtaining the $\mathbb{R}$-gauging give rise to the massive T-duality of [6].

mass parameters	$\mathrm{SL}(2,\mathbb{R})$
(m_1, m_2, m_3)	triplet
$(m_4, \tilde{m}_4)$	doublet
$(m_{11}, m_{\rm IIA})$	doublet
$m_{\rm IIB}$	singlet

Table 5: This table indicates the different multiplets that the $D = 9$ mass parameters form under $\mathrm{SL}(2,\mathbb{R})$.

All these deformations correspond to a gauging of a 9D global symmetry. In particular, it is always the symmetry that is employed in the SS reduction Ansatz that becomes gauged upon reduction. The corresponding gauge vector is always provided by the metric, i.e. is the Kaluza-Klein vector of the dimensional reduction. In all but one case this is the complete story and one finds an abelian gauged supergravity. It turns out that there is one exception where we find a *non-abelian* gauge symmetry. This can be understood from the following general rule.[10] As we noted, the Kaluza-Klein vector gauges the symmetry employed in the SS reduction Ansatz. The fate of either of the remaining two gauge vectors is restricted to three possibilities:

- The vector is a singlet under the gauge symmetry and its field strength acquires no modification, e.g. $A^{(1)}$ in the $m_{\rm IIA}$ deformation.
- The vector transforms under the gauge symmetry and its field strength acquires a massive deformation proportional to a two-form. The degrees of freedom of the vector are eaten up by the two-form via the Stückelberg mechanism, e.g. A in the $m_{\rm IIA}$ deformation.

[10] We thank Sergio Ferrara for clarifying discussions on this issue.

JHEP10(2002)061

- The vector transforms under the gauge symmetry and its field strength acquires no massive deformation proportional to a two-form. In this case we must have gauge enhancement to preserve covariance, e.g. $A^{(1)}$ in the m_4 deformation.

All cases we find in $D = 9$ are consistent with this rule of thumb. We will discuss the different massive deformations one by one below.

5.1 Deformation m_{IIA}: SS reduction of IIA using $\hat{\alpha}$

We first perform a Scherk-Schwarz reduction of (massless) IIA supergravity based on the $\hat{\alpha}$-symmetry of table 2. We use the reduction Ansätze (C.9) with $m_4 = 0$. This leads to a gauged supergravity with mass parameter m_{IIA}. The explicit massive deformations in this case appear in the variation of the gravitino and one of the dilatinos:

$$m_{\mathrm{IIA}} : \begin{cases} \delta_{m_{\mathrm{IIA}}} \psi_\mu = -\frac{9}{14} i m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma_\mu \epsilon^* \,, \\ \delta_{m_{\mathrm{IIA}}} \tilde{\lambda} = \frac{6}{\sqrt{7}} m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \epsilon \,. \end{cases} \tag{5.5}$$

The implicit massive deformations are given by

$$\begin{gathered} \mathrm{D}\phi = e^{-\phi} \mathrm{d} e^{\phi} \,, \qquad \mathrm{D}\varphi = e^{-\varphi} \mathrm{D} e^{\varphi} \,, \qquad G_1 = \mathrm{d}\chi \,, \qquad G_4 = \mathrm{D}C + \vec{B}^T \eta \vec{F} \,, \\ F = \mathrm{D}A - 3 m_{\mathrm{IIA}} B^{(2)} \,, \qquad F^{(1)} = \mathrm{d}A^{(1)} \,, \qquad F^{(2)} = \mathrm{d}A^{(2)} \,, \\ H^{(1)} = \mathrm{D}B^{(1)} - A F^{(1)} + 3 m_{\mathrm{IIA}} C \,, \qquad H^{(2)} = \mathrm{D}B^{(2)} - A F^{(2)} \,. \end{gathered} \tag{5.6}$$

The $\mathbb{R}^+$-covariant derivative is defined by $\mathrm{D} = \mathrm{d} + w_\alpha \, m_{\mathrm{IIA}} A^{(2)}$ with w_α the α scaling-weight of the field it acts on, as given in the table 4, and $\mathrm{DD} = w_\alpha \, m_{\mathrm{IIA}} F^{(2)}$. The covariant derivative of the supersymmetry parameter is given by

$$D_\mu \epsilon = \left(\partial_\mu + \omega_\mu + \frac{i}{4} e^{\phi} \partial_\mu \chi + \frac{9}{14} m_{\mathrm{IIA}} \Gamma_\mu \not{A}^{(2)} \right) \epsilon \,. \tag{5.7}$$

The 9D fermionic field equations have the following explicit massive deformations:

$$m_{\mathrm{IIA}} : \begin{cases} X_{m_{\mathrm{IIA}}}(\psi^\mu) = i m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^{\mu\nu} [\frac{9}{2} \psi_\nu^* - i \frac{9}{32} \gamma_\nu \lambda + i \frac{3}{4\sqrt{7}} \gamma_\nu \tilde{\lambda}] \,, \\ X_{m_{\mathrm{IIA}}}(\lambda) = -m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu [-i \frac{\sqrt{7}}{6} \gamma_\nu \tilde{\lambda}^*] \,, \\ X_{m_{\mathrm{IIA}}}(\tilde{\lambda}) = -m_{\mathrm{IIA}} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu [\frac{6}{\sqrt{7}} \psi_\nu - \frac{11}{6\sqrt{7}} i \gamma_\nu \lambda^* + \frac{1}{7} i \gamma_\nu \tilde{\lambda}^*] \,. \end{cases}$$

This massive deformation is a gauging of the $\mathbb{R}^+$ symmetry α:

$$\alpha : \qquad \Lambda = e^{w_\alpha m_{\mathrm{IIA}} \lambda^{(2)}} \qquad \text{with} \qquad A^{(2)} \to A^{(2)} - \mathrm{d}\lambda^{(2)} \,, \tag{5.9}$$

where w_α are the weights under α.

5.2 Deformation m_4: SS reduction of IIA using $\hat{\beta}$

We next perform a generalized Scherk-Schwarz reduction of $D = 10$ IIA supergravity using the $\mathbb{R}^+$ $\hat{\beta}$ symmetry of table 2. We use the reduction Ansätze given in eq. (C.9), taken

JHEP10(2002)061

with $m_{\text{IIA}} = 0$. This leads to a massive deformation with mass parameter m_4. Only the supersymmetry variations of the dilatinos receive explicit massive deformations:

$$m_4 : \begin{cases} \delta_{m_4}\lambda = \frac{3}{4} m_4 e^{\phi/2-3\varphi/2\sqrt{7}}\epsilon\,, \\ \delta_{m_4}\tilde{\lambda} = \frac{\sqrt{7}}{4} m_4 e^{\phi/2-3\varphi/2\sqrt{7}}\epsilon\,. \end{cases} \tag{5.10}$$

The implicit massive deformations read:

$$\begin{aligned} &\mathrm{D}\phi = e^{-\phi}\mathrm{D}e^{\phi}\,, \qquad \mathrm{D}\varphi = e^{-\varphi}\mathrm{D}e^{\varphi}\,, \qquad G_1 = \mathrm{D}\chi + \frac{3}{4}m_4 A^{(1)}\,, \qquad G_4 = \mathrm{D}C + \vec{B}^T\eta\vec{F}\,, \\ &F = \mathrm{D}A - \frac{1}{2}m_4 B^{(2)}\,, \qquad F^{(1)} = \mathrm{D}A^{(1)}\,, \qquad F^{(2)} = \mathrm{d}A^{(2)}\,, \\ &H^{(1)} = \mathrm{D}B^{(1)} - AF^{(1)} - \frac{1}{4}m_4(C - 3A^{(1)}B^{(2)})\,, \qquad H^{(2)} = \mathrm{D}B^{(2)} - AF^{(2)}\,. \end{aligned} \tag{5.11}$$

The $\mathbb{R}^+$-covariant derivative is defined by $\mathrm{D} = \mathrm{d} + w_\beta\, m_4 A^{(2)}$ with w_β the β scaling-weight of the field it acts on, as given in the table 4, and $\mathrm{DD} = w_\beta\, m_4 F^{(2)}$. The covariant derivative of the supersymmetry parameter has no massive deformation. The explicit deformations of the fermionic field equations read

$$m_4 : \begin{cases} X_{m_4}(\psi^\mu) = i m_4 e^{\phi/2-3\varphi/2\sqrt{7}}\gamma^{\mu\nu}[-i\frac{3}{256}\gamma_\nu\lambda - i\frac{\sqrt{7}}{256}\gamma_\nu\tilde{\lambda}]\,, \\ X_{m_4}(\lambda) = -m_4 e^{\phi/2-3\varphi/2\sqrt{7}}\gamma^\nu[\frac{3}{4}\psi_\nu + \frac{2}{9\sqrt{7}}i\gamma_\nu\tilde{\lambda}^*]\,, \\ X_{m_4}(\tilde{\lambda}) = -m_4 e^{\phi/2-3\varphi/2\sqrt{7}}\gamma^\nu[\frac{\sqrt{7}}{4}\psi_\nu - \frac{2}{9\sqrt{7}}i\gamma_\nu\lambda^*]\,. \end{cases}$$

These massive deformations can be seen as a gauging of the $\mathbb{R}^+$ symmetry β with gauge parameter β and gauge field transformation

$$\beta : \qquad \Lambda = e^{w_\beta m_4 \beta} \qquad \text{with} \qquad A^{(2)} \to A^{(2)} - \mathrm{d}\beta\,. \tag{5.13}$$

In addition, we find that the parabolic subgroup of $\mathrm{SL}(2,\mathbb{R})$, with parameter ζ, is gauged:

$$\zeta : \begin{array}{c} \chi \to \chi + \frac{3}{4}m_4\zeta\,, \\ B^{(1)} \to B^{(1)} + \frac{3}{4}m_4\zeta B^{(2)}\,, \end{array} \qquad \text{with } A^{(1)} \to A^{(1)} - d\zeta + \frac{3}{4}m_4\zeta A^{(2)}\,. \tag{5.14}$$

These two scaling symmetries do not commute but rather form the two-dimensional non-abelian Lie group A(1), consisting of collinear transformations [25, 26] (scalings and translations) in one dimension. The algebra reads

$$[T_\zeta, T_\beta] = T_\zeta\,, \tag{5.15}$$

which is non-semi-simple.

5.3 Deformations m_1, m_2, m_3: SS reduction of IIB using $\mathrm{SL}(2,\mathbb{R})$

We next perform a Scherk-Schwarz reduction of $D = 10$ IIB supergravity using an abelian subgroup of the $\mathrm{SL}(2,\mathbb{R})$ symmetry. This case has been treated in [6, 9, 29, 20, 24, 18, 21].

JHEP10(2002)061

We use the reduction Ansätze given in eq. (C.14) with $m_{\rm IIB} = 0$. This yields massive deformations in $D = 9$ with mass parameters $\vec{m} = (m_1, m_2, m_3)$. Both the explicit and implicit deformations of the supersymmetry rules can be written in terms of the superpotential

$$W(\phi,\chi,\varphi) = \frac{1}{4}e^{2\varphi/\sqrt{7}}\left(m_2\sinh(\phi) + m_3\cosh(\phi) + m_1 e^{\phi}\chi - \frac{1}{2}(m_2 - m_3)e^{\phi}\chi^2\right) \tag{5.16}$$

and the mass matrix employed in the Scherk-Schwarz reduction

$$\mathcal{M} = \frac{1}{2}\begin{pmatrix} m_1 & m_2 + m_3 \\ m_2 - m_3 & -m_1 \end{pmatrix}. \tag{5.17}$$

The explicit deformations are

$$\vec{m}: \begin{cases} \delta_{\vec{m}}\psi_\mu = \frac{1}{7}\gamma_\mu W\epsilon\,, \\ \delta_{\vec{m}}\lambda = 4i(\frac{\delta W}{\delta\phi} + ie^{-\phi}\frac{\delta W}{\delta\chi})\epsilon^*\,, \\ \delta_{\vec{m}}\tilde{\lambda} = 4i\frac{\delta W}{\delta\varphi}\epsilon^*\,, \end{cases} \tag{5.18}$$

while the implicit massive deformations read

$$\begin{aligned} \mathrm{D}\tau &= \mathrm{d}\tau + 4e^{-2\varphi/\sqrt{7}-\phi}\left(\frac{\delta W}{\delta\phi} + ie^{-\phi}\frac{\delta W}{\delta\chi}\right)A \\ &= \left(\mathrm{d} + \frac{1}{2}[(m_2+m_3)\tau^{-1} + 2m_1 + (m_3 - m_2)\tau]A\right)\tau\,, \\ F &= \mathrm{d}A\,, \qquad \vec{F} = \mathrm{d}\vec{A} - \mathcal{M}\vec{B}\,, \qquad \vec{H} = \mathrm{d}\vec{B} - A\vec{F}\,, \\ G_4 &= \mathrm{d}C + \vec{B}^T\eta\vec{F} + \frac{1}{2}\vec{B}^T\eta\mathcal{M}\vec{B}\,, \end{aligned} \tag{5.19}$$

for the bosons and

$$D_\mu\epsilon = \left(\partial_\mu + \omega_\mu + \frac{i}{4}e^{\phi}\partial_\mu\chi - ie^{-2\varphi/\sqrt{7}}WA_\mu\right)\epsilon \tag{5.20}$$

for the supersymmetry parameter. The field equations of the 9D fermions receive the following explicit massive corrections:

$$\vec{m}: \begin{cases} X_{\vec{m}}(\psi^\mu) = -\gamma^{\mu\nu}\left[W\psi_\nu - \frac{1}{16}i\left(\frac{\delta W}{\delta\phi} + ie^{-\phi}\frac{\delta W}{\delta\chi}\right)\gamma_\nu\lambda^* - \frac{1}{16}i\frac{\delta W}{\delta\varphi}\gamma_\nu\tilde{\lambda}^*\right], \\ X_{\vec{m}}(\lambda) = -i\gamma^\mu\left[4\left(\frac{\delta W}{\delta\phi} + ie^{-\phi}\frac{\delta W}{\delta\chi}\right)\psi^*_\mu - \frac{1}{3}iW\gamma_\mu\lambda - \frac{8}{9\sqrt{7}}i\left(\frac{\delta W}{\delta\phi} + ie^{-\phi}\frac{\delta W}{\delta\chi}\right)\gamma_\mu\tilde{\lambda}\right], \\ X_{\vec{m}}(\tilde{\lambda}) = -i\gamma^\nu\left[4\frac{\delta W}{\delta\varphi}\psi^*_\nu - \frac{8}{9\sqrt{7}}i\left(\frac{\delta W}{\delta\phi} - ie^{-\phi}\frac{\delta W}{\delta\chi}\right)\gamma_\nu\lambda - \frac{1}{7}iW\gamma_\nu\tilde{\lambda}\right]. \end{cases}$$

The massive deformations with parameters $\vec{m} = (m_1, m_2, m_3)$ gauge a subgroup of the global $\mathrm{SL}(2,\mathbb{R})$ symmetry (5.3) with parameter and gauge field transformation:

$$\mathrm{SL}(2,\mathbb{R}): \qquad \Omega = e^{\mathcal{M}\lambda}\,, \quad \text{with } A \to A - \mathrm{d}\lambda\,, \qquad \vec{B} \to \Omega(\vec{B} - \vec{A}\,\mathrm{d}\lambda)\,. \tag{5.21}$$

Thus these massive deformations correspond to the gauging of the subgroup of $\mathrm{SL}(2,\mathbb{R})$ with generator $\mathcal{M}$, the mass matrix employed in the reduction. Note that the transformations of this subgroup have special properties: for example, the superpotential W is invariant under it. We can distinguish three distinct cases depending on the value of $\vec{m}^2 = \frac{1}{4}({m_1}^2 + {m_2}^2 - {m_3}^2)$ [23, 24]:

- $\vec{m}^2 = 0$. We gauge an $\mathbb{R}$ subgroup of $\mathrm{SL}(2,\mathbb{R})$ with parameter ζ and invariant metric $\mathrm{diag}(0,1)$.
- $\vec{m}^2 > 0$. We gauge an $\mathrm{SO}(1,1)^+$ subgroup of $\mathrm{SL}(2,\mathbb{R})$ with parameter γ and invariant metric $\mathrm{diag}(1,-1)$.
- $\vec{m}^2 < 0$. We gauge an $\mathrm{SO}(2)$ subgroup of $\mathrm{SL}(2,\mathbb{R})$ with parameter θ and invariant metric $\mathrm{diag}(1,1)$.

All these three cases are one-parameter massive deformations.

5.4 Deformation m_{IIB}: SS reduction of IIB using $\hat{\delta}$

Next, we perform a Scherk-Schwarz reduction of $D = 10$ IIB supergravity using the $\mathbb{R}^+$ symmetry $\hat{\delta}$ of table 3. We use the reduction Ansätze given in eq. (C.14) with $m_1 = m_2 = m_3 = 0$. This yields a massive deformation with parameter m_{IIB}. The explicit deformations of the supersymmetry rules read

$$m_{\mathrm{IIB}}: \begin{cases} \delta_{m_{\mathrm{IIB}}} \psi_\mu = -\tfrac{4}{7} i m_{\mathrm{IIB}} e^{2\varphi/\sqrt{7}} \gamma_\mu \epsilon \,, \\ \delta_{m_{\mathrm{IIB}}} \tilde{\lambda} = -\tfrac{4}{\sqrt{7}} m_{\mathrm{IIB}} e^{2\varphi/\sqrt{7}} \epsilon^* \,. \end{cases} \tag{5.22}$$

The implicit deformations read

$$\begin{aligned} F &= \mathrm{d}A \,, \qquad \vec{F} = \mathrm{d}\vec{A} - 2m_{\mathrm{IIB}}\vec{B} \,, \qquad \vec{H} = \mathrm{d}\vec{B} - A\vec{F} \,, \\ G_4 &= \mathrm{d}C + \vec{B}^T \eta \vec{F} + m_{\mathrm{IIB}} \vec{B}^T \eta \vec{B} \,, \qquad D\varphi = \mathrm{d}\varphi - \frac{4}{\sqrt{7}} m_{\mathrm{IIB}} A \,, \end{aligned} \tag{5.23}$$

for the bosons and

$$D_\mu \epsilon = \left(\partial_\mu + \omega_\mu + \frac{i}{4} e^\phi \partial_\mu \chi + \frac{4}{7} m_{\mathrm{IIB}} \Gamma_\mu \not{A} \right) \epsilon \tag{5.24}$$

for the supersymmetry parameter. The explicit deformations of the fermionic field equations read

$$m_{\mathrm{IIB}}: \begin{cases} X_{m_{\mathrm{IIB}}}(\psi^\mu) = i m_{\mathrm{IIB}} e^{2\varphi/\sqrt{7}} \gamma^{\mu\nu} \left[4\psi_\nu - \tfrac{15}{16\sqrt{7}} i \gamma_\nu \tilde{\lambda}^* \right] , \\ X_{m_{\mathrm{IIB}}}(\lambda) = m_{\mathrm{IIB}} e^{2\varphi/\sqrt{7}} \gamma^\nu \left[\tfrac{4}{9} i \gamma_\nu \lambda \right] , \\ X_{m_{\mathrm{IIB}}}(\tilde{\lambda}) = m_{\mathrm{IIB}} e^{2\varphi/\sqrt{7}} \gamma^\nu \left[\tfrac{4}{\sqrt{7}} \psi_\nu^* - i \tfrac{4}{7} \gamma_\nu \tilde{\lambda} \right] . \end{cases} \tag{5.25}$$

This is a supergravity where the $\mathbb{R}^+$-symmetry δ has been gauged:

$$\delta: \qquad \Lambda = e^{w_\delta m_{\mathrm{IIB}} \lambda} \qquad \text{with} \qquad A \to A - \mathrm{d}\lambda \,, \qquad \vec{B} \to e^{2m_{\mathrm{IIB}}\lambda} (\vec{B} - \vec{A}\,\mathrm{d}\lambda) \,. \tag{5.26}$$

5.5 Deformation m_{11}: KK reduction of IIA with m_{11}-deformation

Finally, one can also consider the Kaluza-Klein reduction of the $D = 10$ gauged supergravity discussed in subsection 3.1 (see also figure 1). This leads to a $D = 9$ gauged supergravity with the following explicit deformations

$$m_{11}: \begin{cases} \delta_{m_{11}} \psi_\mu = \tfrac{9}{14} i m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \tau \gamma_\mu \epsilon^* \,, \\ \delta_{m_{11}} \tilde{\lambda} = -\tfrac{6}{\sqrt{7}} m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \tau^* \epsilon \,. \end{cases} \tag{5.27}$$

The bosonic implicit deformations read

$$
\begin{aligned}
&\mathrm{D}\varphi = \mathrm{d}\varphi - \frac{6}{\sqrt{7}} m_{11} A^{(1)}\,, \qquad F = \mathrm{D}A + 3m_{11} B^{(1)}\,, \qquad G_4 = \mathrm{D}C + \vec{B}^T \eta \vec{F}\,, \\
&H^{(1)} = \mathrm{D}B^{(1)} - AF^{(1)}\,, \qquad H^{(2)} = \mathrm{D}B^{(2)} - AF^{(2)} + 3m_{11} C\,,
\end{aligned}
\tag{5.28}
$$

with the $\mathbb{R}^+$-covariant derivative of a field with weight w defined by $\mathrm{D} = \mathrm{d} - w_\alpha m_{11} A^{(1)}$. For the supersymmetry parameter we find

$$
D_\mu \epsilon = \left(\partial_\mu + \omega_\mu + \frac{i}{4} e^\phi \partial_\mu \chi + \frac{9}{14} m_{11} \Gamma_\mu \not{A}^{(1)} \right) \epsilon\,. \tag{5.29}
$$

The fermionic field equations are deformed by the massive contributions

$$
m_{11} : \begin{cases}
X_{m_{11}}(\psi^\mu) = -i m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^{\mu\nu} \left[\frac{9}{2} \tau \psi^*_\nu - i \frac{9}{32} \tau^* \gamma_\nu \lambda + i \frac{3}{4\sqrt{7}} \tau \gamma_\nu \tilde{\lambda} \right], \\
X_{m_{11}}(\lambda) = m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu \left[-i\tau \frac{\sqrt{7}}{6} \gamma_\nu \tilde{\lambda}^* \right], \\
X_{m_{11}}(\tilde{\lambda}) = m_{11} e^{\phi/2 - 3\varphi/2\sqrt{7}} \gamma^\nu \left[\frac{6}{\sqrt{7}} \tau^* \psi_\nu - \frac{11}{6\sqrt{7}} i \tau \gamma_\nu \lambda^* + \frac{1}{7} i \tau^* \gamma_\nu \tilde{\lambda}^* \right].
\end{cases}
\tag{5.30}
$$

This massive deformation is a gauging of the $\mathbb{R}^+$ symmetry α:

$$
\alpha : \qquad \Lambda = e^{-w_\alpha m_{11} \lambda^{(1)}} \qquad \text{with} \qquad A^{(1)} \to A^{(1)} - \mathrm{d}\lambda^{(1)}\,. \tag{5.31}
$$

This reduction does not lead to a new gauged supergravity. It differs from the m_{IIA} case in the order of the reductions from $D = 11$. In case 1 one first performs an ordinary KK reduction and next a SS reduction on $\hat{\alpha}$ while in the present case the order of these reductions is reversed: one first performs a SS reduction on $\hat{\hat{\alpha}}$ and next an ordinary KK reduction. Indeed, the difference is just a field redefinition via S-duality plus a relabelling of the mass parameters: $m_{11} = m_{\mathrm{IIA}}$. The two mass parameters $(m_{11}, m_{\mathrm{IIA}})$ form a doublet under more general $\mathrm{SL}(2,\mathbb{R})$ field redefinitions.

6. Combining massive deformations

In the previous section we have constructed seven gauged supergravities, each containing a single mass parameter. In this section we would like to consider combining the massive deformations discussed in the previous section. The resulting theories will have more mass parameters characterizing the different deformations. However, not all combinations will turn out to be consistent with supersymmetry. This inconsistency only appears when turning to the bosonic field equations: the supersymmetry algebra with a combination of massive deformations always closes, as can be seen from the following argument.

Suppose one has a supergravity with one massive deformation m and supersymmetry transformations $\delta_0 + \delta_m$. In all cases discussed in this paper the massive deformation of the supersymmetry rules satisfies the following property: $\delta_m(\text{boson}) = 0$. In other words, only the supersymmetry variations of the fermions receive massive corrections. This implies that the issue of the closure of the supersymmetry algebra is a calculation with

JHEP10(2002)061

m-independent parts and parts linear in m but no parts of higher order in m.[11] On the one hand $[\delta(\epsilon_1), \delta(\epsilon_2)]$ has no terms quadratic in m since one of the two δ's acts on a boson. On the other hand the supersymmetry algebra closes modulo fermionic field equations which also have only terms independent of and linear in m. Therefore, given the closure of the massless algebra, the closure of the massive supersymmetry algebra only requires the cancellation of terms linear in m.

In the previous sections we have not checked the closure of the massive supersymmetry algebras since this was guaranteed by the higher-dimensional origin, i.e. Scherk-Schwarz reduction of supergravity leads to a gauged supergravity. However, the argument of linearity allows us to combine different massive deformations. Suppose one has two massive supersymmetry algebras with transformations $\delta_0 + \delta_{m_a}$ and $\delta_0 + \delta_{m_b}$. Both supersymmetry algebras close modulo fermionic field equations with (different) massive deformations. Then the combined massive algebra with transformation $\delta_0 + \delta_{m_a} + \delta_{m_b}$ also closes modulo fermionic field equations whose massive deformations are given by the sum of the separate massive deformations linear in m_a and m_b. The closure of the combined algebra is guaranteed by the closure of the two massive algebras since it requires a cancellation at the linear level.

Under supersymmetry variation of the fermionic field equations, one in general finds linear *and* quadratic deformations of the bosonic equations of motion. In addition to these corrections, we find that there are also 'non-dynamical' equations posing constraints on the mass parameters. Solving these equations generically excludes the possibility of combining massive deformations by requiring mass parameters to vanish. At first sight, one might seem surprised that the supersymmetry variation of the fermionic equations of motion leads to constraints other than the bosonic field equations. However, one should keep in mind that the multiplets involved cannot be linearized around a Minkowski vacuum solution. Therefore, the usual rules for linearized (Minkowski) multiplets do not apply here.

We find that generically adding massive deformations is possible whenever the $D = 10$ symmetries, giving rise to the separate massive deformations, can be combined in $D = 10$ as symmetries of IIA or IIB supergravity only. The combined $D = 9$ supergravity is then a gauged supergravity which just follows by performing a SS reduction on the combined $D = 10$ symmetry.

As a warming-up exercise we will in the first subsection discuss the situation in $D = 10$. In the next subsection we will review the $D = 9$ situation.

6.1 Combining massive deformations in 10D

The 10D IIA supergravity theory has two massive deformations parameterized by m_{R} and m_{11}. Can we combine these two massive deformations? Based on the linearity argument presented above one would expect a closed supersymmetry algebra. The bosonic field equations (with up to quadratic deformations) can be derived by applying the supersymmetry transformations (with only linear deformations) to the fermionic field equations (containing

[11] That is, up to cubic order in fermions. We have not checked the higher-order fermionic terms but, based upon dimensional arguments, we do not expect that these rule out the possibility of combining massive deformations.

only linear deformations). For simplicity, we truncate all bosonic fields to zero except the metric and the dilaton. We thus find

$$
\begin{aligned}
&(\delta_0+\delta_{m_{\rm R}}+\delta_{m_{11}})(X_0+X_{m_{\rm R}}+X_{m_{11}})(\hat\psi^{\hat\mu})=\\
&=\frac{1}{2}\hat\Gamma^{\hat\nu}\hat\epsilon\left[\hat R^{\hat\mu}{}_{\hat\nu}-\frac{1}{2}\hat R\hat g^{\hat\mu}{}_{\hat\nu}-\frac{1}{2}(\partial^{\hat\mu}\hat\phi)(\partial_{\hat\nu}\hat\phi)+\frac{1}{4}(\partial\hat\phi)^2\hat g^{\hat\mu}{}_{\hat\nu}+\frac{1}{4}m_{\rm R}^2e^{5\hat\phi/2}\hat g^{\hat\mu}{}_{\hat\nu}+36m_{11}^2e^{-3\hat\phi/2}\hat g^{\hat\mu}{}_{\hat\nu}\right]+\\
&\quad+\Gamma_{11}\hat\epsilon\left[3m_{11}e^{-3\hat\phi/4}\partial^{\hat\mu}\hat\phi\right]+\Gamma_{11}\hat\Gamma^{\hat\mu}\hat\epsilon\left[\frac{15}{4}m_{\rm R}m_{11}e^{\hat\phi/2}\right],\\
&(\delta_0+\delta_{m_{\rm R}}+\delta_{m_{11}})(X_0+X_{m_{\rm R}}+X_{m_{11}})(\hat\lambda)=\\
&=\hat\epsilon\left[\Box\hat\phi-\frac{5}{4}m_{\rm R}^2e^{5\hat\phi/2}\right]+\hat\Gamma^{\hat\nu}\Gamma_{11}\hat\epsilon\left[9m_{11}e^{-3\hat\phi/4}\partial_{\hat\nu}\hat\phi\right]+\Gamma_{11}\hat\epsilon\left[\frac{33}{2}m_{\rm R}m_{11}e^{\hat\phi/2}\right].
\end{aligned}
\tag{6.1}
$$

At the right-hand side we find four different structures. Three of them correspond to the field equations of the metric, dilaton and RR vector. The vector field equation correspond to the terms linear in m_{11} and containing Γ_{11}. They show us that truncating the RR vector to zero forces us to further truncate the dilaton to $\phi=c$. More interesting is the fourth structure which is bilinear in $m_{\rm R}m_{11}$. It leads to the constraint $m_{\rm R}m_{11}=0$. This constraint cannot be a remnant of a higher-rank form field equation due to its lack of Lorentz indices. It could only fit in the scalar field equation but the Γ_{11} factor prevents this. It is an extra constraint which does not restrict degrees of freedom but rather restricts mass parameters.

We conclude that, even though the closure of the algebra is a linear calculation and therefore always works for combinations, the bosonic field equations exclude the possibility of the combination of massive deformations in $D=10$ dimensions.

6.2 Combining massive deformations in 9D

We next try to combine massive deformations in nine dimensions. One might hope that, due to the large amount of mass parameters, the bosonic field equations do not exclude all possible combinations, like in $D=10$. For the present purposes we will focus on specific terms in the supersymmetry variations of the fermionic field equations. In the following δ_m and X_m are understood to mean the supersymmetry variation and fermionic field equation at linear order containing the sum of all seven possible massive deformations derived in the previous section. Variation of the fermionic field equations gives, amongst other γ-structures, the terms

$$
\begin{aligned}
(\delta_0+\delta_m)(X_0+X_m)(\psi^\mu)&\sim i\,\gamma^\mu\epsilon[\ldots]+\gamma^\mu\epsilon^*[\ldots]+i\,\gamma^\mu\epsilon^*[\ldots],\\
(\delta_0+\delta_m)(X_0+X_m)(\lambda)&\sim\epsilon[\ldots]+i\,\epsilon[\ldots],\\
(\delta_0+\delta_m)(X_0+X_m)(\tilde\lambda)&\sim\epsilon[\ldots]+i\,\epsilon[\ldots]+\epsilon^*[\ldots],
\end{aligned}
\tag{6.2}
$$

where the $[\ldots]$ denote different bosonic real expressions of mass bilinears and scalar factors. These are the analog of the ten-dimensional expression $[m_{\rm R}m_{11}e^{\hat\phi/2}]$ we encountered in the previous subsection. They are the sources for possible constraints on the mass parameters. Requiring all expressions $[\ldots]$ to vanish one is led to the following possible combinations (with the other mass parameters vanishing):

JHEP10(2002)061

- **Case 1** with $\{m_{\rm IIA}, m_4\}$: this combination can also be obtained by Scherk-Schwarz reduction of IIA employing a linear combination of the symmetries $\hat{\alpha}$ and $\hat{\beta}$, guaranteeing its consistency. It is also a gauging of both this symmetry and (for $m_4 \neq 0$) the parabolic subgroup of $\mathrm{SL}(2, \mathbb{R})$ in 9D, giving the non-abelian gauge group A(1).

- **Case 2,3,4** with $\{\vec{m}, m_{\rm IIB}\}$: as in the case with $m_{\rm IIB} = 0$ and only $\vec{m}$ this combination contains three different, inequivalent cases depending on $\vec{m}^2$ (depending crucially on the fact that $m_{\rm IIB}$ is a singlet under $\mathrm{SL}(2, \mathbb{R})$):

  - **Case 2** with $\{\vec{m}, m_{\rm IIB}\}$ and $\vec{m}^2 = 0$.
  - **Case 3** with $\{\vec{m}, m_{\rm IIB}\}$ and $\vec{m}^2 > 0$.
  - **Case 4** with $\{\vec{m}, m_{\rm IIB}\}$ and $\vec{m}^2 < 0$.

  All these combinations can also be obtained by Scherk-Schwarz reduction of IIB employing a linear combination of the symmetries $\hat{\delta}$ and (one of the subgroups of) $\mathrm{SL}(2, \mathbb{R})$, guaranteeing its consistency. All cases (assuming that $m_{\rm IIB} \neq 0$) correspond to the gauging of an abelian non-compact symmetry in 9D. Only the special case $\{\vec{m}^2 < 0, m_{\rm IIB} = 0\}$ corresponds to an SO(2)-gauging.

- **Case 5** with $\{m_4 = -\frac{12}{5} m_{\rm IIA}, m_2 = m_3\}$: this case can be understood as the generalized dimensional reduction of Romans' massive IIA theory, employing the $\mathbb{R}^+$ symmetry that is not broken by the $m_{\rm R}$ deformations: $\hat{\beta} - \frac{5}{12}\hat{\alpha}$. It gauges both this linear combination of $\mathbb{R}^+$'s and the parabolic subgroup of $\mathrm{SL}(2, \mathbb{R})$ in 9D, giving the non-abelian gauge group A(1).

Another solution to the quadratic constraints has parameters $\{m_{\rm IIA}, m_{11}\}$, but this combination does not represent a new case. It can be obtained from only $m_{\rm IIA}$ (and thus a truncation of Case 1) via an $\mathrm{SL}(2, \mathbb{R})$ field redefinition (since they form a doublet). Thus the most general deformations are the five cases given above, all containing two mass parameters. All five of these are gauged theories and have a higher-dimensional origin. Both case 1 and case 5 have a non-abelian gauge group provided $m_4 \neq 0$.

7. Solutions

In the first part of this paper we constructed a variety of gauged supergravities with 32 supersymmetries. They all have in common that there is a scalar potential. Our next goal is to make a systematic search for solutions that are based on this scalar potential. In the next subsections we will search for two types of solutions: (i) 1/2 BPS domain-wall (DW) solutions and (ii) maximally symmetric solutions with constant scalars, i.e. de Sitter (dS), Minkowski (Mink) or anti-de Sitter (AdS) solutions.

7.1 1/2 BPS DW solutions

In our previous paper [18] we already made a systematic search for half-supersymmetric DW solutions of the gauged supergravities corresponding to the cases 3, 4 and 5. Due to

JHEP10(2002)061

a one-to-one relationship with 7-branes in $D = 10$ dimensions [29] we could even make a systematic investigation of the quantization of the mass parameters by using the results of [31, 32].

The goal of this subsection is to investigate whether the five massively deformed supergravities we found in subsection 6.2 allow new half-supersymmetric DW solutions. In other words, we will derive all 1/2 BPS 7-brane solutions to the 9-dimensional supergravities described in the previous sections. This analysis should lead, as a check of our calculations, to at least all the solutions of [18]. Since we are looking for 1/2 BPS solutions it is convenient to solve the Killing spinor equations, which are obtained by setting the supersymmetry variation of the gravitino and dilatinos to zero. In this way we solve first order equations instead of second order equations which we would encounter if we would solve the field equations directly. For static configurations a solution to the Killing spinor equation is also a solution to the field equations, so we don't have to explicitly check that the field equations are satisfied. The projector[12] for a DW is given by $\frac{1}{2}(1 \pm \gamma_y)$, where y denotes the transverse direction. We find that, in order to make a projection operator in the Killing spinor equations, we are forced to set all mass parameters to zero except for $\vec{m}$, which corresponds to cases 3, 4 and 5 of section 6. This is a consistent combination of masses and we obtain three classes of domain wall solution which were discussed in detail in [18]. As it turns out, there are no more half-supersymmetric DW solutions.

To summarize, we find that there are no new codimension-one 1/2 BPS solutions to the $D = 9$ supergravity theories we obtained in the previous sections, as compared to the three classes of domain wall solutions given in [18].

7.2 Solutions with constant scalars

In this subsection we will consider solutions with all three scalars constant. This is a consistent truncation in two cases which both have two mass parameters. In this truncation one is left with the metric only satisfying the Einstein equation with a cosmological term

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} \, , \tag{7.1}$$

with Λ quadratic in the two mass parameters. Depending on the sign of this term one thus has anti-de Sitter, Minkowski or de Sitter geometry.

We find that solutions with constant scalars are possible in the following massive supergravities:

- $\boldsymbol{D = 10}$ with $\{m_{11}\}$ has $\Lambda = 36 {m_{11}}^2 e^{-3\hat{\phi}/2}$, which gives rise to de Sitter$_{10}$ [9], breaking all supersymmetry. The $D = 11$ origin of this solution is Mink$_{11}$ written in a basis where the x-dependence is of the required form [9]:

$$\text{Mink}_{11}: \qquad ds^2 = e^{2m_{11}x}\left(-dt^2 + e^{2m_{11}t} dx_9^2 + dx^2\right) . \tag{7.2}$$

[12] From a general analysis of the possible projectors in 9 dimensions, i.e. demanding that the projector squares to itself and that its trace is half of the spinor dimension, in order to yield a 1/2 BPS state, we find that there is a second projector given by $\frac{1}{2}(1 \pm i\gamma_t)$. This projector would give a euclidean DW, i.e. a DW having time as a transverse direction. Note that such a euclidean DW can never be 1/2 BPS since if there existed a Killing spinor it would square to a Killing vector in the *transverse* direction, i.e. time, which is not an isometry of the euclidean DW.

- $\boldsymbol{D} = \mathbf{9}$, **Case 1** with $\{m_{\rm IIA} = -\frac{2}{3}m_4\}$ has $\Lambda = \frac{63}{4}{m_4}^2 e^{\phi - 3\varphi/\sqrt{7}}$, which gives rise to De Sitter$_9$, breaking all supersymmetry. This case follows from the reduction of Mink$_{10}$ by using a combination of IIA scale symmetries that leave the dilaton invariant (since Minkowski has vanishing dilaton) so that, after reduction, one is left with a non-trivial geometry only.

- $\boldsymbol{D} = \mathbf{9}$, **Case 4** with $\{m_{\rm IIB}, m_3\}$ has $\Lambda = 28{m_{\rm IIB}}^2 e^{4\varphi/\sqrt{7}}$, which gives rise to de Sitter$_9$ for non-vanishing $m_{\rm IIB}$. This case follows from the reduction of Mink$_{10}$ by using a combination of IIB scale symmetries that leave the dilaton invariant. Note that for vanishing $m_{\rm IIB}$ this reduces to Mink$_9$, despite the presence of m_3 [20]. For either $m_{\rm IIB}$ or m_3 non-zero this solution breaks all supersymmetry.

8. Conclusions

In this paper we have constructed five different $D = 9$ massive deformations with 32 supersymmetries, each containing two mass parameters. All these five theories have a higher-dimensional origin via SS reduction from $D = 10$ dimensions. Furthermore, the massive deformations gauge a global symmetry of the massless theory. The gauge groups we have obtained are the abelian groups SO(2), SO$(1,1)^+$, $\mathbb{R}$, $\mathbb{R}^+$ and the unique two-dimensional non-abelian Lie group A(1) of scalings and translations on the real line.

We have analyzed the possibility of combining massive deformations to obtain more general massive supergravities that are not gauged or do not have a higher-dimensional origin. Our analysis shows that the only possible combinations are the five two-parameter deformations, which are all gauged and can be uplifted. We have not made a systematic search for massive $D = 9$ supergravities that are not the combination of gaugings and we cannot exclude that there are more possibilities. This requires a separate calculation. In this context, it is of interest to point out that examples of massive supergravities like Romans have been found in lower dimensions, e.g. [33, 34]. In these cases the compactification manifolds are such that the candidate gauge fields are truncated away.

It is intriguing that some of the gauged supergravities we have constructed result from gauging an $\mathbb{R}^+$ scale symmetry that does not leave the lagrangian invariant but scales it with a factor. Apparently, it is possible to gauge such symmetries at the level of the equations of motion. It would be interesting to work out the general procedure for doing this.

We now would like to address the question of whether the gauged supergravities we constructed can be interpreted as the leading terms in a low-energy approximation to (compactified) superstring theory. Let us first discuss the status of the $D = 10$ gauged supergravity. There exist two ways in the literature to construct this theory:

1. In [8] the theory was constructed by pointing out that the Bianchi identities of $D = 11$ superspace allow a more general solution involving a conformal spin connection. This more general solution is equivalent to standard $D = 11$ supergravity for a topologically trivial spacetime but leads to a new possibility for a nontrivial spacetime

of the form $M_{10} \times S^1$. The reduction over the circle leads to the $D = 10$ gauged supergravity theory.

2. In [9] the same $D = 10$ gauged supergravity was obtained via SS reduction of the standard $D = 11$ supergravity using the $\mathbb{R}^+$ scale symmetry of the $D = 11$ equations of motion.

In both cases it is not obvious how to extend the reduction procedure beyond the lowest order approximation. The higher-order derivative terms which arise as corrections in M-theory seem to break the scale invariance of the $D = 11$ equations of motion.[13] The symmetry used to reduce is therefore only a symmetry of the lowest order approximation. Presumably this means that the more general procedure of [8] involving the conformal spin connection also does not work in the presence of higher-order corrections.

One could try to restore the scale invariance by treating the $D = 11$ Planck length $\ell_{\rm p}$ or, equivalently, the $D = 11$ Einstein constant κ, as a scalar field $\ell_{\rm p}(x)$ and giving it a nontrivial weight under the scale transformations. This can be done by adding to the $D = 11$ lagrangian a Lagrange multiplier term of the form

$$\Delta\mathcal{L} = \int d^{11}x\ \ell_{\rm p}(x)\, d\Lambda^{(10)}\,, \tag{8.1}$$

where $\ell_{\rm p}(x)$ is the x-dependent $D = 11$ Planck length and $\Lambda^{(10)}$ is a 10-form Lagrange multiplier field.[14] The problem of the above approach is that, after SS reduction, one is left with *two* Lagrange multiplier fields. The field equation for one of them implies that the string parameter $\ell_{\rm s}$ is a constant. The other field equation, however, leads to the constraint that $m_{11}\ell_{\rm s} = 0$. Thus, one should take either $m_{11} = 0$ or $\ell_{\rm s} = 0$. In the first case there is no deformation left while in the second case one is forced to consider string theory in the $\ell_{\rm s} = 0$ limit where no higher-order corrections survive. Naturally, the scale symmetry survives in this limit.

However, the fact that the gauged 10D supergravity with mass parameter m_{11} does not seem to have a higher dimensional origin in the presence of higher derivative corrections does not exclude a possible rôle for it in string theory. In this sense its status is similar to Romans' massive theory which also can not be obtained from 11D supergravity plus corrections. Of course the difference is that Romans' theory has a well understood string theory origin which is lacking for the m_{11} theory.

The same discussion carries over to nine dimensions. The massive deformations split up in two categories: those where only the theory to lowest order in α' has a higher-dimensional origin and those where also the higher-derivative corrections can be obtained from 10D. The latter category can be derived using symmetries that extend to all orders in α'. We have two such symmetries:

[13] We thank Shamit Kachru and Neil Lambert for a discussion on this point.

[14] A similar procedure can be performed at the level of the Green-Schwarz action of the $D = 11$ supermembrane [35] where the membrane tension is replaced by a worldvolume 2-form potential. This introduces a scale symmetry in the Green-Schwarz action. In fact, one can show that in the formulation of [35] the Green-Schwarz action is invariant under the same scale transformations that leave the equations of motion of $D = 11$ supergravity invariant.

- The $\mathrm{SL}(2,\mathbb{R})$ (or rather its $\mathrm{SL}(2,\mathbb{Z})$ subgroup) symmetry of IIB. Thus the $\vec{m} = (m_1, m_2, m_3)$ deformations correspond to the low-energy limits of three different sectors of compactified IIB string theory (depending on $\vec{m}^2 = \frac{1}{4}({m_1}^2 + {m_2}^2 - {m_3}^2)$). In [18] DW solutions were constructed for all three sectors. Of these only the D7-brane has a well-understood role in IIB string theory.

- The linear combination $\frac{1}{12}\alpha + \beta$ of $\mathbb{R}^+$-symmetries of IIA. Thus one can define a massive deformation m_s within Case I with $\{m_{\mathrm{IIA}} = \frac{1}{12}m_s, m_4 = m_s\}$ which corresponds to the low-energy limit of a sector of compactified IIA string theory. No vacuum solution has been constructed for this sector. It would be very interesting to try to find a vacuum solution and understand which role it plays in IIA string theory.

In fact, one can have a better understanding of the m_s massive deformation and the $\frac{1}{12}\alpha + \beta$ symmetry of IIA from the following point of view. The combination $\frac{1}{12}\alpha + \beta$ of IIA can be understood from its 11D origin as the general coordinate transformation $x^{11} \to \lambda\, x^{11}$. This explains why all α' corrections transform covariantly under this specific $\mathbb{R}^+$: the higher-order corrections in 11D are invariant under general coordinate transformations and upon reduction they must transform covariantly under the reduced g.c.t.'s, among which is the $\frac{1}{12}\alpha + \beta$ scaling-symmetry.

The transformation $x^{11} \to \lambda\, x^{11}$ can also be used for a Scherk-Schwarz reduction from 11D to 9D with a different procedure to give internal coordinate dependence to the fields. Let us call this an SS2 reduction as opposed to the SS1 reduction, which is the method we have used throughout the paper and which is based on *global, internal symmetries* of the higher-dimensional theory. The SS2 procedure [36] instead uses *a symmetry of the compactification manifold* for the reduction Ansatz.[15] The massive deformations resulting from a SS2 reduction can be expressed in terms of the structure constants of the corresponding non-abelian gauge group. Using the transformation $x^{11} \to \lambda\, x^{11}$ in the SS2 reduction from 11D to 9D we obtain massive deformations which are equal to the m_s deformations upon relating the components of ${f_{ab}}^c$ to m_s. Indeed, this explains why the m_s deformations correspond to a gauging of the 2D non-abelian Lie group A(1) rather than only the $\mathbb{R}^+$ symmetry $\frac{1}{12}\alpha + \beta$.

The understanding of the m_s deformation in terms of a SS2 reduction employing $x^{11} \to \lambda\, x^{11}$ also explains why $\tilde{m}_4$ cannot be obtained from a SS1 reduction. Since S-duality interchanges x^{10} and x^{11}, it is the g.c.t. $x^{10} \to \lambda\, x^{10}$ that would give rise to a $m_{11} = \frac{1}{12}\tilde{m}_4$ deformation. However, this transformation is not an internal symmetry of 10D IIA supergravity and thus cannot be exploited in a SS1 reduction. Since m_{11} does have a 10D origin, this implies that $\tilde{m}_4$ cannot be obtained from 10D IIA.

The $D = 9$ gauged supergravities involving m_{11}, m_{IIB} or $m_{\mathrm{IIA}} \neq \frac{1}{12}m_4$ have the same status as the $D = 10$ gauged supergravity discussed above, i.e. these theories are based upon symmetries that are broken by α'-corrections. Note that all the de Sitter space solutions we found in section 7 involve either m_{11}, m_{IIB} or $m_{\mathrm{IIA}} \neq \frac{1}{12}m_4$. It would be

[15] It was already noted by Scherk and Schwarz that SS1 reduction with a symmetry that originates from a higher-dimensional g.c.t. is equivalent to the corresponding SS2 reduction. For an example, see [37].

interesting to see whether these de Sitter spaces could occur as the $\ell_s \to 0$ limit of an exact solution of string theory.

Acknowledgments

We are grateful to Jisk Attema, Sergio Ferrara, Rein Halbersma, Shamit Kachru, Neil Lambert, Jan Louis, Tomás Ortín, Jan Pieter van der Schaar, Kelly Stelle and Paul Townsend for interesting and useful discussions. This work is supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime, in which the University of Groningen is associated with the University of Utrecht. The work of T.d.W. and U.G. is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM). The work of R.L. is supported by the Mexico National Council for Science and Technology (CONACyT) under grant 010085.

A. Conventions

We use mostly plus signature $(- + \cdots +)$. All metrics are Einstein-frame metrics. Doubly hatted fields and indices are eleven-dimensional, singly hatted fields and indices ten-dimensional while unhatted ones are nine-dimensional. Greek indices $\hat{\mu}, \hat{\nu}, \hat{\rho} \ldots$ denote world coordinates and Latin indices $\hat{a}, \hat{b}, \hat{c} \ldots$ represent tangent spacetime. They are related by the Vielbeins $\hat{e}_{\hat{\mu}}{}^{\hat{a}}$ and inverse Vielbeins $\hat{e}_{\hat{a}}{}^{\hat{\mu}}$. Explicit indices x, y are underlined when flat and non-underlined when curved. When indices are omitted we use form notation.

B. Scherk-Schwarz reduction of dilaton-gravity

In this appendix we will discuss in detail the most general Scherk-Schwarz reduction of the dilaton-gravity system.

We start with the truncation of 10D IIA and IIB supergravity to the metric and the dilaton. The lagrangian reads

$$\hat{\mathcal{L}} = \sqrt{-\hat{g}} \left[\hat{R} - \frac{1}{2}(\partial \hat{\phi})^2 \right] , \tag{B.1}$$

while the corresponding Euler-Lagrange equations are given by

$$\begin{aligned} [\hat{g}^{\hat{\mu}\hat{\nu}}] : &\qquad \hat{R}_{\hat{\mu}\hat{\nu}} - \frac{1}{2}\hat{R}\hat{g}_{\hat{\mu}\hat{\nu}} - \frac{1}{2}\partial_{\hat{\mu}}\hat{\phi}\partial_{\hat{\nu}}\hat{\phi} + \frac{1}{4}(\partial\hat{\phi})^2 \hat{g}_{\hat{\mu}\hat{\nu}} = 0 \, , \\ [\hat{\phi}] : &\qquad \Box\hat{\phi} = 0 \, . \end{aligned} \tag{B.2}$$

This system has two global symmetries: one can either scale the metric or one can shift the dilaton:

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to e^{2m_g} \hat{g}_{\hat{\mu}\hat{\nu}} \, , \qquad \hat{\phi} \to \hat{\phi} + m_\phi \, . \tag{B.3}$$

The shift of the dilaton is a symmetry of the lagrangian. The scale transformation of the metric is a symmetry of the field equations only; it scales the lagrangian. This will prove an important difference when performing Scherk-Schwarz reductions. We will show that one has to reduce field equations, rather than the lagrangian, when performing SS reductions with symmetries of the field equations only.

Using an arbitrary linear combination of the two global symmetries we make the following Ansatz for Scherk-Schwarz reduction over x to nine dimensions:

$$\hat{e}_{\hat{\mu}}{}^{\hat{a}} = e^{m_g x} \begin{pmatrix} e^{\sqrt{7}\varphi/28} e_\mu{}^a & 0 \\ 0 & e^{-\sqrt{7}\varphi/4} \end{pmatrix}, \qquad \hat{\phi} = \phi + m_\phi x\,, \tag{B.4}$$

where we have omitted the Kaluza-Klein vector A_μ for simplicity. Using this Ansatz the 10D field equations yield the following 9D equations:

$$\begin{aligned} [\hat{g}^{\mu\nu}]: \qquad & R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}(\partial\phi)^2 g_{\mu\nu} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{4}(\partial\varphi)^2 g_{\mu\nu} + \\ & + e^{4\varphi/\sqrt{7}}\left(\frac{1}{4}{m_\phi}^2 + 28{m_g}^2\right) g_{\mu\nu} = 0\,, \\ [\hat{\phi}]: \qquad & \Box\phi + 8 m_g m_\phi e^{4\varphi/\sqrt{7}} = 0\,, \\ [\hat{g}^{xx}]: \qquad & \Box\varphi - \frac{2}{\sqrt{7}}{m_\phi}^2 e^{4\varphi/\sqrt{7}} = 0\,. \end{aligned} \tag{B.5}$$

Note that the field equations of the metric and both scalars get bilinear massive deformations. In addition one has the reduction of the $\hat{g}^{x\mu}$ field equation

$$[\hat{g}^{x\mu}]: \qquad 2\sqrt{7} m_g \partial_\mu\varphi + \frac{1}{2} m_\phi \partial_\mu \phi = 0\,, \tag{B.6}$$

which is the equation of motion for the Kaluza-Klein vector A_μ. Since it is not important for our argument we will not consider this equation and restrict to (B.5). We will discuss whether this sector of the field equations can be reproduced by a lagrangian.

If one performs the SS reduction on the 10D lagrangian, instead of on the field equations, the result reads $\hat{\mathcal{L}} = e^{8 m_g x}\mathcal{L}$ with the 9D lagrangian given by

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\varphi)^2 - V(\phi,\varphi)\right] \quad \text{with} \quad V(\phi,\varphi) = e^{4\varphi/7}\left(\frac{1}{2}{m_\phi}^2 + 72{m_g}^2\right). \tag{B.7}$$

The corresponding Euler-Lagrange equations read

$$\begin{aligned} [g^{\mu\nu}]: \qquad & R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}(\partial\phi)^2 g_{\mu\nu} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{4}(\partial\varphi)^2 g_{\mu\nu} + \\ & + e^{4\varphi/\sqrt{7}}\left(\frac{1}{4}{m_\phi}^2 + 36{m_g}^2\right) g_{\mu\nu} = 0\,, \\ [\phi]: \qquad & \Box\phi = 0\,, \\ [\varphi]: \qquad & \Box\varphi - \frac{4}{\sqrt{7}} e^{4\varphi/\sqrt{7}}\left(\frac{1}{2}{m_\phi}^2 + 72{m_g}^2\right) = 0\,. \end{aligned} \tag{B.8}$$

These Euler-Lagrange equations only coincide with the reduction of the 10D Euler-Lagrange equations (B.5) provided $m_g = 0$. Thus the application of SS reduction to the lagrangian does not give the correct answer if the lagrangian scales: the Euler-Lagrange equations (B.8) are not equal to the field equations (B.5) for $m_g \neq 0$.[16] In fact, the situation is worse [9]: for $m_g \neq 0$ there *is* no lagrangian $\mathcal{L}$ with potential $V(\phi,\varphi)$ whose Euler-Lagrange equations are the correct field equations (B.5). The metric field equation would require

$$V(\phi,\varphi) = e^{4\varphi/\sqrt{7}} \left(\frac{1}{2} {m_\phi}^2 + 56 {m_g}^2 \right) , \tag{B.9}$$

but this is inconsistent with the ϕ and φ field equations for $m_g \neq 0$.

Thus we conclude that Scherk-Schwarz reduction on the lagrangian is only legitimate when the exploited symmetry leaves the lagrangian *invariant* rather than covariant. For symmetries that scale the lagrangian one has to reduce the field equations. Including the full field content, such as the Kaluza-Klein vector A_μ, does not change this conclusion. One could hope to improve the situation by first going to a frame in which the metric is invariant (possible for any m_ϕ, m_g with $m_\phi \neq 0$) and then do the SS reduction. Since this is related by a field redefinition it will not change the essential properties: the higher-dimensional lagrangian still scales and the lower-dimensional field equations do not have a corresponding lagrangian.

C. Supergravities and their reductions

C.1 $D = 11$ supergravity

The supersymmetry transformation rules of $N = 1$ eleven-dimensional supergravity read

$$\begin{aligned}
\delta \hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}} &= \bar{\hat{\hat{\epsilon}}} \hat{\hat{\Gamma}}^{\hat{\hat{a}}} \hat{\hat{\psi}}_{\hat{\hat{\mu}}} \,, \\
\delta \hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}} &= -3\, \bar{\hat{\hat{\epsilon}}} \hat{\hat{\Gamma}}_{[\hat{\hat{\mu}}\hat{\hat{\nu}}} \hat{\hat{\psi}}_{\hat{\hat{\rho}}]} \,, \\
\delta \hat{\hat{\psi}}_{\hat{\hat{\mu}}} &= D_{\hat{\hat{\mu}}} \hat{\hat{\epsilon}} + \frac{1}{192} \left(\hat{\hat{\Gamma}}^{(4)} \hat{\hat{\Gamma}}_{\hat{\hat{\mu}}} - \frac{1}{3} \hat{\hat{\Gamma}}_{\hat{\hat{\mu}}} \hat{\hat{\Gamma}}^{(4)} \right) \hat{\hat{G}}_{(4)} \hat{\hat{\epsilon}} \,,
\end{aligned} \tag{C.1}$$

with the field strengths $\hat{\hat{G}}_{(4)} = \mathrm{d}\hat{\hat{C}}$ and $D_{\hat{\hat{\mu}}} \hat{\hat{\epsilon}} = (\partial_{\hat{\hat{\mu}}} + \hat{\hat{\omega}}_{\hat{\hat{\mu}}}) \hat{\hat{\epsilon}}$. The 11D fermionic field content consists solely of a 32-component gravitino, whose field equation reads

$$X_0(\hat{\hat{\psi}}^{\hat{\hat{\mu}}}) \equiv \hat{\hat{\Gamma}}^{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}} \hat{\hat{D}}_{\hat{\hat{\nu}}} \hat{\hat{\psi}}_{\hat{\hat{\rho}}} = 0 \,, \tag{C.2}$$

with $\hat{\hat{D}}_{\hat{\hat{\nu}}} = \partial_{\hat{\hat{\nu}}} + \hat{\hat{\omega}}_{\hat{\hat{\nu}}}$ and where we have set the three-form equal to zero. Under supersymmetry this fermionic field equations transforms into

$$\delta_0 X_0(\hat{\hat{\psi}}^{\hat{\hat{\mu}}}) = \frac{1}{2} \hat{\hat{\Gamma}}^{\hat{\hat{\nu}}} \hat{\hat{\epsilon}} \left[\hat{\hat{R}}^{\hat{\hat{\mu}}}{}_{\hat{\hat{\nu}}} - \frac{1}{2} \hat{\hat{R}} \hat{\hat{g}}^{\hat{\hat{\mu}}}{}_{\hat{\hat{\nu}}} \right] , \tag{C.3}$$

which implies the bosonic Einstein equation for the metric.

[16] The difference between substitution in the lagrangian or its field equations, in a slightly different context, was also discussed in [38].

We use the following reduction Ansätze

$$
\begin{aligned}
\hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}} &= e^{m_{11}x}\begin{pmatrix} e^{-\hat{\phi}/12}\hat{e}_{\hat{\mu}}{}^{\hat{a}} & -e^{2\hat{\phi}/3}\hat{A}_{\hat{\mu}} \\ 0 & e^{2\hat{\phi}/3}\end{pmatrix}\,,\\
\hat{\hat{\psi}}_{\hat{a}} &= e^{-m_{11}x/2}e^{\hat{\phi}/24}\left[\hat{\psi}_{\hat{a}} - \frac{1}{24}\Gamma_{\hat{a}}\hat{\lambda}\right],\\
\hat{\hat{\psi}}_{\underline{x}} &= \frac{1}{3}e^{-m_{11}x/2}e^{\hat{\phi}/24}\Gamma_{\underline{x}}\hat{\lambda}\,,\\
\hat{\hat{\epsilon}} &= e^{m_{11}x/2}e^{-\hat{\phi}/24}\hat{\epsilon}\,,\\
\hat{\hat{C}}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= e^{3m_{11}x}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}\,, \qquad \hat{\hat{C}}_{\hat{\mu}\hat{\nu}x} = -e^{3m_{11}x}\hat{B}_{\hat{\mu}\hat{\nu}}\,,
\end{aligned} \tag{C.4}
$$

to arrive at the IIA susy-rules in ten dimensions.

C.2 $D = 10$ IIA supergravity

The supersymmetry transformation rules of ten-dimensional IIA supergravity read

$$
\begin{aligned}
\delta_0\hat{e}_{\hat{\mu}}{}^{\hat{a}} &= \bar{\hat{\epsilon}}\Gamma^{\hat{a}}\hat{\psi}_{\hat{\mu}}\,,\\
\delta_0\hat{\psi}_{\hat{\mu}} &= \Big(D_{\hat{\mu}} + \frac{1}{48}e^{-\hat{\phi}/2}\Big(\hat{\not{H}}\hat{\Gamma}_{\hat{\mu}} + \frac{1}{2}\hat{\Gamma}_{\hat{\mu}}\hat{\not{H}}\Big)\Gamma_{11} + \frac{1}{16}e^{3\hat{\phi}/4}\Big(\hat{\not{F}}\hat{\Gamma}_{\hat{\mu}} - \frac{3}{4}\hat{\Gamma}_{\hat{\mu}}\hat{\not{F}}\Big)\Gamma_{11} + \\
&\quad + \frac{1}{192}e^{\hat{\phi}/4}\Big(\hat{\not{G}}\hat{\Gamma}_{\hat{\mu}} - \frac{1}{4}\hat{\Gamma}_{\hat{\mu}}\hat{\not{G}}\Big)\Big)\hat{\epsilon}\,,\\
\delta_0\hat{B}_{\hat{\mu}\hat{\nu}} &= 2e^{\hat{\phi}/2}\bar{\hat{\epsilon}}\Gamma_{11}\hat{\Gamma}_{[\hat{\mu}}\left(\hat{\psi}_{\hat{\nu}]} + \frac{1}{8}\hat{\Gamma}_{\hat{\nu}]}\hat{\lambda}\right),\\
\delta_0\hat{A}_{\hat{\mu}} &= -\,e^{-3\hat{\phi}/4}\bar{\hat{\epsilon}}\Gamma_{11}\left(\hat{\psi}_{\hat{\mu}} - \frac{3}{8}\hat{\Gamma}_{\hat{\mu}}\hat{\lambda}\right),\\
\delta_0\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= -\,3e^{-\hat{\phi}/4}\bar{\hat{\epsilon}}\hat{\Gamma}_{[\hat{\mu}\hat{\nu}}\left(\hat{\psi}_{\hat{\rho}]} - \frac{1}{24}\hat{\Gamma}_{\hat{\rho}]}\hat{\lambda}\right) + 3\hat{A}_{[\hat{\mu}}\delta_0\hat{B}_{\hat{\nu}\hat{\rho}]}\,,\\
\delta_0\hat{\lambda} &= \left(\hat{\not{\partial}}\hat{\phi} + \frac{1}{12}e^{-\hat{\phi}/2}\hat{\not{H}}\Gamma_{11} + \frac{3}{8}e^{3\hat{\phi}/4}\hat{\not{F}}\Gamma_{11} + \frac{1}{96}e^{\hat{\phi}/4}\hat{\not{G}}\right)\hat{\epsilon}\,,\\
\delta_0\hat{\phi} &= \frac{1}{2}\bar{\hat{\epsilon}}\hat{\lambda}\,,
\end{aligned} \tag{C.5}
$$

with the following field strengths:

$$
\hat{F} = \mathrm{d}\hat{A}\,, \qquad \hat{H} = \mathrm{d}\hat{B}\,, \qquad \hat{G} = \mathrm{d}\hat{C} + \hat{A}\hat{H}\,, \tag{C.6}
$$

and $D_{\hat{\mu}}\hat{\epsilon} = (\partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}})\hat{\epsilon}$. Upon (massless) reduction with our Ansätze the 11D field equation splits up in two field equations for the 10D IIA fermionic field content, a gravitino and a dilatino:

$$
\begin{aligned}
X_0(\hat{\psi}^{\hat{\mu}}) &\equiv \hat{\Gamma}^{\hat{\mu}\hat{\nu}\hat{\rho}}\hat{D}_{\hat{\nu}}\hat{\psi}_{\hat{\rho}} - \frac{1}{8}(\hat{\not{\partial}}\hat{\phi})\hat{\Gamma}^{\hat{\mu}}\hat{\lambda} = 0\,,\\
X_0(\hat{\lambda}) &\equiv \hat{\Gamma}^{\hat{\nu}}\hat{D}_{\hat{\nu}}\hat{\lambda} - \hat{\Gamma}^{\hat{\nu}}(\hat{\not{\partial}}\hat{\phi})\hat{\psi}_{\hat{\nu}} = 0\,,
\end{aligned} \tag{C.7}
$$

with $\hat{D}_{\hat{\nu}} = (\partial_{\hat{\nu}} + \hat{\omega}_{\hat{\nu}})$ and where we have set the vector, two- and three-form equal to zero. Under supersymmetry these fermionic field equations transform into

$$\begin{aligned}
\delta_0 X_0(\hat{\psi}^{\hat{\mu}}) &= \frac{1}{2}\hat{\Gamma}^{\hat{\nu}}\hat{\epsilon}\left[\hat{R}^{\hat{\mu}}{}_{\hat{\nu}} - \frac{1}{2}\hat{R}\hat{g}^{\hat{\mu}}{}_{\hat{\nu}} - \frac{1}{2}(\partial^{\hat{\mu}}\hat{\phi})(\partial_{\hat{\nu}}\hat{\phi}) + \frac{1}{4}(\partial\hat{\phi})^2\hat{g}^{\hat{\mu}}{}_{\hat{\nu}}\right], \\
\delta_0 X_0(\hat{\lambda}) &= \hat{\epsilon}\,[\Box\hat{\phi}]\,,
\end{aligned} \tag{C.8}$$

which imply the usual graviton-dilaton field equations.

We use the following reduction Ansatz with x-dependence implied by the $\mathbb{R}^+$-symmetries $\hat{\alpha}$ and $\hat{\beta}$, given in table 2:

$$\begin{aligned}
\hat{e}_{\hat{\mu}}{}^{\hat{a}} &= e^{9m_{\rm IIA}x/8}\begin{pmatrix} e^{\phi/16-3\varphi/16\sqrt{7}}e_\mu{}^a & -e^{-7\phi/16+3\sqrt{7}\varphi/16}A_\mu^{(2)} \\ 0 & e^{-7\phi/16+3\sqrt{7}\varphi/16}\end{pmatrix}, \\
\hat{\psi}_a &= e^{-9m_{\rm IIA}x/16}e^{-\phi/32+3\varphi/32\sqrt{7}}\left[\psi_a + \frac{1}{32}\Gamma_a\left(\lambda - \frac{3}{\sqrt{7}}\tilde{\lambda}\right)\right], \\
\hat{\psi}_{\underline{x}} &= -\frac{7}{32}e^{-9m_{\rm IIA}x/16}e^{-\phi/32+3\varphi/32\sqrt{7}}\Gamma_{\underline{x}}\left(\lambda - \frac{3}{\sqrt{7}}\tilde{\lambda}\right), \\
\hat{B}_{\mu\nu} &= e^{3m_{\rm IIA}x+m_4x/2}(B^{(2)}_{\mu\nu} - 2A^{(2)}_{[\mu}A_{\nu]})\,, \\
\hat{B}_{\mu x} &= -e^{3m_{\rm IIA}x+m_4x/2}A_\mu\,, \\
\hat{A}_\mu &= -e^{-3m_4x/4}(A^{(1)}_\mu - \chi A^{(2)})\,, \\
\hat{A}_x &= -e^{-3m_4x/4}\chi\,, \\
\hat{C}_{\mu\nu\rho} &= e^{3m_{\rm IIA}x-m_4x/4}\left(C_{\mu\nu\rho} - 3A^{(1)}_{[\mu}B^{(2)}_{\nu\rho]} + 3A^{(2)}_{[\mu}B^{(1)}_{\nu\rho]} + 6A^{(1)}_{[\mu}A^{(2)}_{\nu}A_{\rho]}\right), \\
\hat{C}_{\mu\nu x} &= -e^{3m_{\rm IIA}x-m_4x/4}(B^{(1)}_{\mu\nu} - 2A^{(1)}_{[\mu}A_{\nu]})\,, \\
\hat{\lambda} &= \frac{1}{4}e^{-9m_{\rm IIA}x/16}e^{-\phi/32+3\varphi/32\sqrt{7}}(3\lambda + \sqrt{7}\tilde{\lambda})\,, \\
\hat{\epsilon} &= e^{9m_{\rm IIA}x/16}e^{\phi/32-3\varphi/32\sqrt{7}}\epsilon\,, \\
\hat{\phi} &= \frac{1}{4}(3\phi + \sqrt{7}\varphi) + \left(\frac{3}{2}m_{\rm IIA} + m_4\right)x\,.
\end{aligned} \tag{C.9}$$

C.3 $D = 10$ IIB supergravity

The supersymmetry transformation rules of ten-dimensional IIB supergravity read (in complex notation)

$$\begin{aligned}
\delta\hat{e}_{\hat{\mu}}{}^{\hat{a}} &= \frac{1}{2}\bar{\hat{\epsilon}}\hat{\Gamma}^{\hat{a}}\hat{\psi}_{\hat{\mu}} + \text{h.c.}\,, \\
\delta\hat{\psi}_{\hat{\mu}} &= D_{\hat{\mu}}\hat{\epsilon} - \frac{i}{16\cdot 5!}\not{\hat{G}}^{(5)}\hat{\Gamma}_{\hat{\mu}}\hat{\epsilon} + \frac{i}{16\cdot 3!}e^{\hat{\phi}/2}\left(\hat{\Gamma}_{\hat{\mu}}\hat{\Gamma}^{(3)} + 2\hat{\Gamma}^{(3)}\hat{\Gamma}_{\hat{\mu}}\right)\left(\hat{H}^{(1)} - \hat{\tau}\hat{H}^{(2)}\right)_{(3)}\hat{\epsilon}^*\,, \\
\delta\hat{\lambda} &= -e^{\hat{\phi}}\not{\partial}\hat{\tau}\hat{\epsilon}^* - \frac{1}{2\cdot 3!}e^{\hat{\phi}/2}\hat{\Gamma}^{(3)}\left(\hat{H}^{(1)} - \hat{\tau}\hat{H}^{(2)}\right)_{(3)}\hat{\epsilon}\,, \\
\delta\hat{B}^{(1)}_{\hat{\mu}\hat{\nu}} &= -e^{\hat{\phi}/2}\hat{\tau}^*\left(\bar{\hat{\epsilon}}^*\hat{\Gamma}_{[\hat{\mu}}\hat{\psi}_{\hat{\nu}]} - \frac{i}{8}\bar{\hat{\epsilon}}\hat{\Gamma}_{\hat{\mu}\hat{\nu}}\hat{\lambda}\right) + \text{h.c.}\,, \\
\delta\hat{B}^{(2)}_{\hat{\mu}\hat{\nu}} &= -e^{\hat{\phi}/2}\left(\bar{\hat{\epsilon}}^*\,\hat{\Gamma}_{[\hat{\mu}}\hat{\psi}_{\hat{\nu}]} - \frac{i}{8}\bar{\hat{\epsilon}}\hat{\Gamma}_{\hat{\mu}\hat{\nu}}\hat{\lambda}\right) + \text{h.c.}\,,
\end{aligned}$$

$$
\begin{aligned}
\delta\hat{D}_{\hat{\mu}\hat{\nu}\hat{\lambda}\hat{\rho}} &= 2i\,\bar{\hat{\epsilon}}\,\hat{\Gamma}_{[\hat{\mu}\hat{\nu}\hat{\lambda}}\hat{\psi}_{\hat{\rho}]} - \frac{3}{2}\,\varepsilon_{ij}\hat{B}^{(i)}_{[\hat{\mu}\hat{\nu}}\delta\hat{B}^{(j)}_{\hat{\lambda}\hat{\rho}]} + \text{h.c.}\,, \\
\delta\hat{\chi} &= -\frac{1}{4}e^{-\hat{\phi}}\bar{\hat{\epsilon}}\hat{\lambda}^* + \text{h.c.}\,, \\
\delta\hat{\phi} &= \frac{i}{4}\bar{\hat{\epsilon}}\hat{\lambda}^* + \text{h.c.}\,,
\end{aligned} \tag{C.10}
$$

with the complex scalar $\hat{\tau} = \hat{\chi} + ie^{-\hat{\phi}}$ and the field strengths

$$
\vec{\hat{H}} = \mathrm{d}\vec{\hat{B}}\,, \qquad \hat{G} = \mathrm{d}\hat{D} + \frac{1}{2}\vec{\hat{B}}^T\eta\vec{\hat{H}}\,, \qquad \eta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{C.11}
$$

The field strength $\hat{G}$ is subject to a self-duality constraint. The covariant derivative of the IIB Killing spinor reads

$$
D_{\hat{\mu}}\hat{\epsilon} = \left(\partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}} + \frac{i}{4}e^{\hat{\phi}}\partial_{\hat{\mu}}\hat{\chi}\right)\hat{\epsilon}\,. \tag{C.12}
$$

When truncating to the metric, scalars and fermions, the massless 10D IIB fermionic field equations read

$$
\begin{aligned}
X_0(\hat{\psi}^{\hat{\mu}}) &\equiv \hat{\Gamma}^{\hat{\mu}\hat{\nu}\hat{\rho}}\left(\partial_{\hat{\nu}} + \hat{\omega}_{\hat{\nu}} + \frac{1}{4}ie^{\hat{\phi}}\partial_{\hat{\nu}}\hat{\chi}\right)\hat{\psi}_{\hat{\rho}} + \frac{1}{8}e^{\hat{\phi}}(\not{\partial}\hat{\tau})\hat{\Gamma}^{\hat{\mu}}\hat{\lambda}^* = 0\,, \\
X_0(\hat{\lambda}) &\equiv \hat{\Gamma}^{\hat{\mu}}\left(\partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}} + \frac{3}{4}ie^{\hat{\phi}}\partial_{\hat{\mu}}\hat{\chi}\right)\hat{\lambda} + e^{\hat{\phi}}\hat{\Gamma}^{\hat{\mu}}(\not{\partial}\hat{\tau})\hat{\psi}^*_{\hat{\mu}} = 0\,.
\end{aligned} \tag{C.13}
$$

The reduction Ansätze we used for reducing the above rules are

$$
\begin{aligned}
\hat{e}_{\hat{\mu}}{}^{\hat{a}} &= e^{m_{\mathrm{IIB}}x}\begin{pmatrix} e^{\sqrt{7}\varphi/28}e_\mu{}^a & -e^{-\sqrt{7}\varphi/4}A_\mu \\ 0 & e^{-\sqrt{7}\varphi/4} \end{pmatrix}, \\
\hat{\psi}_a &= e^{-m_{\mathrm{IIB}}x/2}e^{-\sqrt{7}\varphi/56}\left(\frac{c\tau^*+d}{c\tau+d}\right)^{1/4}\left(\psi_a + \frac{1}{8\sqrt{7}}\Gamma_a\tilde{\lambda}^*\right), \\
\hat{\psi}_{\underline{x}} &= -\frac{\sqrt{7}}{8}e^{-m_{\mathrm{IIB}}x/2}e^{-\sqrt{7}\varphi/56}\left(\frac{c\tau^*+d}{c\tau+d}\right)^{1/4}\Gamma_{\underline{x}}\tilde{\lambda}^*\,, \\
\hat{\lambda} &= ie^{-m_{\mathrm{IIB}}x/2}e^{-\sqrt{7}\varphi/56}\left(\frac{c\tau^*+d}{c\tau+d}\right)^{3/4}\lambda\,, \\
\hat{\epsilon} &= e^{m_{\mathrm{IIB}}x/2}e^{\sqrt{7}\varphi/56}\left(\frac{c\tau^*+d}{c\tau+d}\right)^{1/4}\epsilon\,, \\
\hat{\tau} &= \frac{a\tau+b}{c\tau+d}\,, \\
\vec{\hat{B}}_{\mu\nu} &= e^{2m_{\mathrm{IIB}}x}\Omega(x)\vec{B}_{\mu\nu}\,, \qquad \vec{\hat{B}}_{\mu x} = -e^{2m_{\mathrm{IIB}}x}\Omega(x)\vec{A}_\mu\,, \\
\hat{D}_{\mu\nu\lambda\rho} &= e^{4m_{\mathrm{IIB}}x}D_{\mu\nu\lambda\rho}\,, \qquad \hat{D}_{\mu\nu\lambda x} = e^{4m_{\mathrm{IIB}}x}\left(-C_{\mu\nu\lambda} + \frac{3}{2}\vec{A}^T_{[\mu}\eta\vec{B}_{\nu\rho]}\right),
\end{aligned} \tag{C.14}
$$

where we take the Ω to be x-dependent:

$$
\Omega(x) = \begin{pmatrix} \cosh(\alpha x) + \frac{m_1}{2\alpha}\sinh(\alpha x) & \frac{1}{2\alpha}(m_2+m_3)\sinh(\alpha x) \\ \frac{1}{2\alpha}(m_2-m_3)\sinh(\alpha x) & \cosh(\alpha x) - \frac{m_1}{2\alpha}\sinh(\alpha x) \end{pmatrix}. \tag{C.15}
$$

Upon reduction to 9D the self-duality constraint relates $C_{\mu\nu\lambda}$ to $D_{\mu\nu\lambda\rho}$ and can be used to eliminate the latter.

C.4 $D = 9$ $N = 2$ supergravity

The unique nine-dimensional $N = 2$ supergravity theory has the following supersymmetry transformations:

$$
\begin{aligned}
\delta_0 e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu + \text{h.c.}\,,\\
\delta_0\psi_\mu &= D_\mu\epsilon + \frac{i}{16}e^{-2\varphi/\sqrt{7}}\left(\frac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)F_{(2)}\epsilon-\\
&\quad - \frac{1}{8\cdot 2!}e^{3\varphi/2\sqrt{7}}\left(\frac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)\, e^{\phi/2}\left(F^{(1)} - \tau F^{(2)}\right)_{(2)}\epsilon^* +\\
&\quad + \frac{i}{8\cdot 3!}e^{-\varphi/2\sqrt{7}}\left(\frac{3}{7}\gamma_\mu\gamma^{(3)} + \gamma^{(3)}\gamma_\mu\right) e^{\phi/2}\left(H^{(1)} - \tau H^{(2)}\right)_{(3)}\epsilon^* -\\
&\quad - \frac{1}{8\cdot 4!}e^{\varphi/\sqrt{7}}\left(\frac{1}{7}\gamma_\mu\gamma^{(4)} - \gamma^{(4)}\gamma_\mu\right)G_4\epsilon\,,\\
\delta_0\tilde{\lambda} &= i\not{\partial}\varphi\,\epsilon^* - \frac{1}{\sqrt{7}}e^{-2\varphi/\sqrt{7}}\not{F}\epsilon^* - \frac{3i}{2\cdot 2!\sqrt{7}}e^{3\varphi/2\sqrt{7}}e^{\phi/2}\gamma^{(2)}\left(F^{(1)} - \tau^* F^{(2)}\right)_{(2)}\epsilon\\
&\quad + \frac{1}{2\cdot 3!\sqrt{7}}e^{-\varphi/2\sqrt{7}}e^{\phi/2}\gamma^{(3)}\left(H^{(1)} - \tau^* H^{(2)}\right)_{(3)}\epsilon +\\
&\quad + \frac{i}{4!\sqrt{7}}e^{\varphi/\sqrt{7}}\not{G}_4\epsilon^*\,,\\
\delta_0\lambda &= i\not{\partial}\phi\,\epsilon^* - e^{\phi}\not{\partial}\chi\,\epsilon^* - \frac{i}{2\cdot 2!}e^{3\sqrt{7}\varphi/14}e^{\phi/2}\gamma^{(2)}\left(F^{(1)} - \tau F^{(2)}\right)_{(2)}\epsilon-\\
&\quad - \frac{1}{2\cdot 3!}e^{-\sqrt{7}\varphi/14}e^{\phi/2}\gamma^{(3)}\left(H^{(1)} - \tau H^{(2)}\right)_{(3)}\epsilon\,,\\
\delta_0 A_\mu &= \frac{i}{2}e^{2\varphi/\sqrt{7}}\bar{\epsilon}(\psi_\mu - \frac{i}{\sqrt{7}}\gamma_\mu\tilde{\lambda}^*) + \text{h.c.}\,,\\
\delta_0 A_\mu^{(1)} &= -\frac{i}{2}e^{\phi/2}\tau^* e^{-3\varphi/2\sqrt{7}}\left(\bar{\epsilon}^*\psi_\mu + \frac{i}{4}\bar{\epsilon}\gamma_\mu\lambda + \frac{3i}{4\sqrt{7}}\bar{\epsilon}^*\gamma_\mu\tilde{\lambda}^*\right) + \text{h.c.}\,,\\
\delta_0 A_\mu^{(2)} &= -\frac{i}{2}e^{\phi/2}e^{-3\varphi/2\sqrt{7}}\left(\bar{\epsilon}^*\psi_\mu + \frac{i}{4}\bar{\epsilon}\gamma_\mu\lambda + \frac{3i}{4\sqrt{7}}\bar{\epsilon}^*\gamma_\mu\tilde{\lambda}^*\right) + \text{h.c.}\,,\\
\delta_0 B_{\mu\nu}^{(1)} &= e^{\phi/2}\tau^* e^{\varphi/2\sqrt{7}}\left(\bar{\epsilon}^*\gamma_{[\mu}\psi_{\nu]} + \frac{i}{8}\bar{\epsilon}\gamma_{\mu\nu}\lambda - \frac{i}{8\sqrt{7}}\bar{\epsilon}^*\gamma_{\mu\nu}\tilde{\lambda}^*\right) - A_{[\mu}\delta_0 A_{\nu]}^{(1)} + \text{h.c.}\,,\\
\delta_0 B_{\mu\nu}^{(2)} &= e^{\phi/2}e^{\varphi/2\sqrt{7}}\left(\bar{\epsilon}^*\gamma_{[\mu}\psi_{\nu]} + \frac{i}{8}\bar{\epsilon}\gamma_{\mu\nu}\lambda - \frac{i}{8\sqrt{7}}\bar{\epsilon}^*\gamma_{\mu\nu}\tilde{\lambda}^*\right) - A_{[\mu}\delta_0 A_{\nu]}^{(2)} + \text{h.c.}\,,\\
\delta_0 C_{\mu\nu\lambda} &= \frac{3}{2}e^{-\varphi/\sqrt{7}}\bar{\epsilon}\gamma_{[\mu\nu}\left(\psi_{\lambda]} + \frac{i}{6\sqrt{7}}\gamma_{\lambda]}\tilde{\lambda}^*\right) - \frac{3}{2}\vec{B}^T_{[\mu\nu}\eta\,\delta_0\vec{A}_{\lambda]} + \text{h.c.}\,,\\
\delta_0\varphi &= -\frac{i}{4}\bar{\epsilon}\tilde{\lambda}^* + \text{h.c.}\,,\\
\delta_0\chi &= \frac{1}{4}e^{-\phi}\bar{\epsilon}\lambda^* + \text{h.c.}\,,\\
\delta_0\phi &= -\frac{i}{4}\bar{\epsilon}\lambda^* + \text{h.c.}\,,
\end{aligned}
\tag{C.16}
$$

with the complex scalar $\tau = \chi + ie^{-\phi}$. The field strengths read

$$G_1 = \mathrm{d}\chi\,, \qquad F = \mathrm{d}A\,, \qquad \vec{F} = \mathrm{d}\vec{A}\,, \qquad \vec{H} = \mathrm{d}\vec{B} - A\vec{F}\,, \qquad G_4 = \mathrm{d}C + \vec{B}^T\eta\vec{F}\,. \tag{C.17}$$

The covariant derivative of the Killing spinor reads

$$D_\mu\epsilon = (\partial_\mu + \omega_\mu + \frac{i}{4}e^{\phi}\partial_\mu\chi)\epsilon\,. \tag{C.18}$$

When truncating to the metric, scalars and fermions, the massless 9D fermionic field equations read

$$\begin{aligned}
X_0(\psi^\mu) &\equiv \gamma^{\mu\nu\rho}\left(\partial_\nu + \omega_\nu + \frac{1}{4}ie^{\phi}\partial_\nu\chi\right)\psi_\rho - \frac{1}{8}e^{\phi}(\not{\partial}\tau)\gamma^\mu\lambda^* + \frac{1}{8}i(\not{\partial}\varphi)\gamma^\mu\tilde{\lambda}^* = 0\,,\\
X_0(\lambda) &\equiv \gamma^\mu\left(\partial_\mu + \omega_\mu + \frac{3}{4}ie^{\phi}\partial_\mu\chi\right)\lambda + e^{\phi}\gamma^\mu(\not{\partial}\tau)\psi^*_\mu = 0\,,\\
X_0(\tilde{\lambda}) &\equiv \gamma^\mu\left(\partial_\mu + \omega_\mu - \frac{1}{4}ie^{\phi}\partial_\mu\chi\right)\tilde{\lambda} - i\gamma^\mu(\not{\partial}\varphi)\psi^*_\mu = 0\,.
\end{aligned} \tag{C.19}$$

Under supersymmetry these yield the variation

$$\begin{aligned}
\delta_0 X_0(\psi^\mu) &= \frac{1}{2}\gamma^\nu\epsilon\Big[R^\mu{}_\nu - \frac{1}{2}Rg^\mu{}_\nu - \frac{1}{2}\left((\partial^\mu\phi)(\partial_\nu\phi) - \frac{1}{2}(\partial\phi)^2 g^\mu{}_\nu\right) -\\
&\quad - \frac{1}{2}e^{2\phi}\left((\partial^\mu\chi)(\partial_\nu\chi) - \frac{1}{2}(\partial\chi)^2 g^\mu{}_\nu\right) - \frac{1}{2}\left((\partial^\mu\varphi)(\partial_\nu\varphi) - \frac{1}{2}(\partial\varphi)^2 g^\mu{}_\nu\right)\Big]\,,\\
\delta_0 X_0(\lambda) &= \epsilon^*[-e^{\phi}(\Box\chi + 2(\partial_\mu\phi)(\partial^\mu\chi))] + i\epsilon^*[\Box\phi - e^{2\phi}(\partial\chi)^2]\,,\\
\delta_0 X_0(\tilde{\lambda}) &= i\epsilon^*[\Box\varphi]\,,
\end{aligned} \tag{C.20}$$

which are the massless bosonic field equations for the metric and the scalars.

D. Spinors and Γ-matrices in ten and nine dimensions

The Γ-matrices in ten ($\Gamma_{\hat{\mu}}$) and nine (γ_μ) dimensions can be chosen to satisfy

$$\Gamma^\dagger_{\hat{\mu}} = \eta_{\hat{\mu}\hat{\mu}}\Gamma_{\hat{\mu}} \qquad \text{and} \qquad \gamma^\dagger_\mu = \eta_{\mu\mu}\gamma_\mu\,, \tag{D.1}$$

respectively. In ten dimensions we can also choose the Γ-matrices to be real, while in nine dimensions they will be purely imaginary, which implies that

$$\Gamma^T_{\hat{\mu}} = \eta_{\hat{\mu}\hat{\mu}}\Gamma_{\hat{\mu}} \qquad \text{and} \qquad \gamma^T_\mu = -\eta_{\mu\mu}\gamma_\mu\,. \tag{D.2}$$

In ten dimensions the minimal spinor is a 32 component Majorana-Weyl spinor with 16 (real) degrees of freedom. With the choice

$$\Gamma_{11} \equiv -\Gamma_{\underline{0}\cdots\underline{9}}\,, \qquad \Gamma_{11} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \tag{D.3}$$

we can write a ten-dimensional Majorana-Weyl spinor as being composed of nine-dimensional, 16 component, Majorana-Weyl spinors according to

$$\psi_+^{MW} = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix}, \qquad \psi_-^{MW} = \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}, \tag{D.4}$$

where ψ_i are nine-dimensional Majorana-Weyl spinors and $+$ or $-$ denotes the chirality of the ten-dimensional spinor. The split of an arbitrary ten-dimensional spinor into two Majorana-Weyl spinors of opposite chirality can of course be done without reference to nine dimensions (through the specific choice of Γ_{11}), but each ten-dimensional Majorana-Weyl spinor will then in general have 32 non-zero components even though it only has 16 degrees of freedom. In order to reduce to nine dimensions we use

$$\Gamma_{11} = \sigma_3 \otimes \mathbb{1}\,, \qquad \Gamma_{\underline{x}} = \sigma_1 \otimes \mathbb{1}\,, \qquad \Gamma_a = \sigma_2 \otimes \gamma_a\,, \tag{D.5}$$

where x is the reduction coordinate and the Pauli matrices are defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{D.6}$$

As mentioned above the nine dimensional γ-matrices are purely imaginary. If we work with a reduction of type IIB, where the two spinors have the same chirality, it may be convenient to introduce complex, nine-dimensional, Weyl spinors according to

$$\psi_c = \psi_1 + i\psi_2\,, \qquad \lambda_c = \lambda_2 + i\lambda_1\,, \tag{D.7}$$

$$\epsilon_c = \epsilon_1 + i\epsilon_2\,, \qquad \tilde\lambda_c = \tilde\lambda_2 + i\tilde\lambda_1\,, \tag{D.8}$$

which in ten-dimensional notation can be written as, e.g.,

$$\psi_+^W = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} + i\begin{pmatrix} \psi_2 \\ 0 \end{pmatrix}. \tag{D.9}$$

If we instead work with a reduction of type IIA the two spinors will have opposite chirality, and can thus be composed into a ten-dimensional Majorana spinor according to

$$\psi^M = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}. \tag{D.10}$$

When working with these non-minimal spinors, which are either just Majorana (ψ_μ^M) or just Weyl (ψ_μ^W) [18], the two formulations are (in nine dimensions) related via

$$\begin{aligned}
\frac{1}{2}(1+\Gamma_{11})\psi_\mu^M &= \mathrm{Re}(\psi_\mu^W)\,, & \frac{1}{2}(1-\Gamma_{11})\psi_\mu^M &= \mathrm{Im}(\Gamma_{\underline{x}}\psi_\mu^W)\,,\\
\frac{1}{2}(1+\Gamma_{11})\lambda^M &= \mathrm{Im}(\Gamma_{\underline{x}}\lambda^W)\,, & \frac{1}{2}(1-\Gamma_{11})\lambda^M &= \mathrm{Re}(\lambda^W)\,,\\
\frac{1}{2}(1+\Gamma_{11})\tilde\lambda^M &= \mathrm{Im}(\Gamma_{\underline{x}}\tilde\lambda^W)\,, & \frac{1}{2}(1-\Gamma_{11})\tilde\lambda^M &= \mathrm{Re}(\tilde\lambda^W)\,,\\
\frac{1}{2}(1+\Gamma_{11})\epsilon^M &= \mathrm{Re}(\epsilon^W)\,, & \frac{1}{2}(1-\Gamma_{11})\epsilon^M &= \mathrm{Im}(\Gamma_{\underline{x}}\epsilon^W)\,,
\end{aligned} \tag{D.11}$$

$$
\begin{tikzcd}
 & \psi^{M}_{32\mathbb{R}} \arrow[dr, "Weyl"] & \\
\psi^{D}_{32\mathbb{C}} \arrow[ur, "Majorana"] \arrow[dr, "Weyl"'] & & \psi^{MW}_{16\mathbb{R}} \\
 & \psi^{W}_{16\mathbb{C}} \arrow[ur, "Majorana"'] &
\end{tikzcd}
$$

Figure 2: Schematic view of how a ten dimensional Dirac spinor can be projected down to a Majorana-Weyl spinor along two different routes. The number of real or complex degrees of freedom for each spinor is also indicated. The relation between the spinors at the intermediate stage (in nine dimensions) is given by D.11.

for positive (ψ^W_μ, ϵ^W) and negative $(\lambda^W, \tilde{\lambda}^W)$ chirality Weyl fermions. With the above mentioned decomposition into nine-dimensional Majorana-Weyl spinors we can write

$$\psi^M_\mu = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \epsilon^M = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}, \quad \lambda^M = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \quad \tilde{\lambda}^M = \begin{pmatrix} \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \end{pmatrix} \tag{D.12}$$

and

$$\psi^W_\mu = \begin{pmatrix} \psi_1 + i\psi_2 \\ 0 \end{pmatrix}, \quad \epsilon^W_\mu = \begin{pmatrix} \epsilon_1 + i\epsilon_2 \\ 0 \end{pmatrix}, \tag{D.13}$$

$$\lambda^W = \begin{pmatrix} 0 \\ \lambda_2 + i\lambda_1 \end{pmatrix}, \quad \tilde{\lambda}^W = \begin{pmatrix} 0 \\ \tilde{\lambda}_2 + i\tilde{\lambda}_1 \end{pmatrix}, \tag{D.14}$$

where the spinors without an M or W superscript are Majorana-Weyl spinors.

The two different routes to obtain Majorana-Weyl spinors are illustrated in figure 2. Note also that it follows from the Clifford algebra and the choice of Γ_{11} that $\Gamma_{\underline{x}}$ is off-diagonal, which is crucial for this construction.

References

[1] L.J. Romans, *Massive $N = 2a$ supergravity in ten-dimensions*, *Phys. Lett.* **B 169** (1986) 374.

[2] I.C.G. Campbell and P.C. West, *$N = 2$ $D = 10$ nonchiral supergravity and its spontaneous compactification*, *Nucl. Phys.* **B 243** (1984) 112.

[3] F. Giani and M. Pernici, *$N = 2$ supergravity in ten-dimensions*, *Phys. Rev.* **D 30** (1984) 325.

[4] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[5] J. Polchinski and E. Witten, *Evidence for heterotic–type-I string duality*, *Nucl. Phys.* **B 460** (1996) 525 [hep-th/9510169].

[6] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Duality of type-II 7-branes and 8-branes*, *Nucl. Phys.* **B 470** (1996) 113 [hep-th/9601150].

[7] E. Bergshoeff, Y. Lozano and T. Ortin, *Massive branes*, *Nucl. Phys.* **B 518** (1998) 363 [hep-th/9712115].

[8] P.S. Howe, N.D. Lambert and P.C. West, *A new massive type-IIA supergravity from compactification*, *Phys. Lett.* **B 416** (1998) 303 [hep-th/9707139].

[9] I.V. Lavrinenko, H. Lu and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, *Class. and Quant. Grav.* **15** (1998) 2239 [hep-th/9710243].

[10] A. Chamblin and N.D. Lambert, *de Sitter space from M-theory*, *Phys. Lett.* **B 508** (2001) 369 [hep-th/0102159].

[11] A. Chamblin and N.D. Lambert, *Zero-branes, quantum mechanics and the cosmological constant*, *Phys. Rev.* **D 65** (2002) 066002 [hep-th/0107031].

[12] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[13] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[14] H.J. Boonstra, K. Skenderis and P.K. Townsend, *The domain wall/QFT correspondence*, *J. High Energy Phys.* **01** (1999) 003 [hep-th/9807137].

[15] K. Behrndt, E. Bergshoeff, R. Halbersma and J.P. van der Schaar, *On domain-wall/QFT dualities in various dimensions*, *Class. and Quant. Grav.* **16** (1999) 3517 [hep-th/9907006].

[16] R. Kallosh, A.D. Linde, S. Prokushkin and M. Shmakova, *Gauged supergravities, de Sitter space and cosmology*, *Phys. Rev.* **D 65** (2002) 105016 [hep-th/0110089].

[17] P.K. Townsend, *Quintessence from M-theory*, *J. High Energy Phys.* **11** (2001) 042 [hep-th/0110072].

[18] E. Bergshoeff, U. Gran and D. Roest, *Type IIB seven-brane solutions from nine-dimensional domain walls*, *Class. and Quant. Grav.* **19** (2002) 4207 [hep-th/0203202].

[19] P.M. Cowdall, *Novel domain wall and minkowski vacua of $D = 9$ maximal* SO(2) *gauged supergravity*, hep-th/0009016.

[20] J. Gheerardyn and P. Meessen, *Supersymmetry of massive $D = 9$ supergravity*, *Phys. Lett.* **B 525** (2002) 322 [hep-th/0111130].

[21] H. Nishino and S. Rajpoot, *Gauged $N = 2$ supergravity in nine-dimensions and domain wall solutions*, *Phys. Lett.* **B 546** (2002) 261 [hep-th/0207246].

[22] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60.

[23] C.M. Hull, *Massive string theories from M-theory and F-theory*, *J. High Energy Phys.* **11** (1998) 027 [hep-th/9811021].

[24] C.M. Hull, *Gauged $D = 9$ supergravities and Scherk-Schwarz reduction*, hep-th/0203146.

[25] T. Frankel, *The geometry of physics*, Cambridge University Press, Cambridge 1997.

[26] R. Gilmore, *Lie groups, Lie algebras, and some of their applications*, John Wiley & Sons 1974.

JHEP10(2002)061

[27] E. Bergshoeff, B. Janssen and T. Ortín, *Solution-generating transformations and the string effective action*, *Class. and Quant. Grav.* **13** (1996) 321 [hep-th/9506156].

[28] H. Nicolai, P.K. Townsend and P. van Nieuwenhuizen, *Comments on eleven-dimensional supergravity*, *Lett. Nuovo Cim.* **30** (1981) 315.

[29] P. Meessen and T. Ortín, *An* $\mathrm{SL}(2,\mathbb{Z})$ *multiplet of nine-dimensional type-II supergravity theories*, *Nucl. Phys.* **B 541** (1999) 195 [hep-th/9806120].

[30] E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, R. Halbersma, S. Vandoren and A. Van Proeyen, *Superconformal* $N=2$, $D=5$ *matter with and without actions*, *J. High Energy Phys.* **10** (2002) 045 [hep-th/0205230].

[31] O. DeWolfe, T. Hauer, A. Iqbal and B. Zwiebach, *Uncovering the symmetries on* (p,q) *7-branes: beyond the Kodaira classification*, *Adv. Theor. Math. Phys.* **3** (1999) 1785 [hep-th/9812028].

[32] O. DeWolfe, T. Hauer, A. Iqbal and B. Zwiebach, *Uncovering infinite symmetries on* (p,q) *7-branes: Kac-Moody algebras and beyond*, *Adv. Theor. Math. Phys.* **3** (1999) 1835 [hep-th/9812209].

[33] K. Behrndt, E. Bergshoeff, D. Roest and P. Sundell, *Massive dualities in six dimensions*, *Class. and Quant. Grav.* **19** (2002) 2171 [hep-th/0112071].

[34] J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys.* **B 635** (2002) 395 [hep-th/0202168].

[35] E. Bergshoeff, L.A.J. London and P.K. Townsend, *Space-time scale invariance and the superp-brane*, *Class. and Quant. Grav.* **9** (1992) 2545 [hep-th/9206026].

[36] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[37] E. Bergshoeff, M. de Roo and E. Eyras, *Gauged supergravity from dimensional reduction*, *Phys. Lett.* **B 413** (1997) 70 [hep-th/9707130].

[38] P.K. Townsend, *World sheet electromagnetism and the superstring tension*, *Phys. Lett.* **B 277** (1992) 285.